Biologists observe that the population of a certain bacteria doubles in size every 3 hours. If the culture began with 1000 bacteria, 3 hrs later there were 2000 bacteria. After another 3 hours there were 4000 bacteria, and so on.

Let \( n = \)the number of bacteria after \( t \) hours, then

The number of bacteria after \( t \) hours is:

A population that experiences exponential growth increases according to the model \( n(t) = n_0e^{rt} \)

- \( n(t) = \)population at time \( t \)
- \( n_0 = \)initial size of the population aka \( n(0) \)
- \( r = \)relative rate of growth
- \( t = \)time

The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per hour.

a. Find a function that models the number of bacteria after \( t \) hours.
b. What is the estimated count after 10 hours?
c. Sketch the graph of the function \( n(t) \).
Example 2: Comparing Effects of Different Rates of Population Growth

In 2000, the population of the world was 6.1 billion people and the relative rate of growth was 1.4% How does this growth rate compare to say 1.0% by the year 2050?

Example 3: A certain breed of rabbit was introduced onto a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100, with a relative growth rate of 55% per year.

What was the initial size of the rabbit population?
Estimate the population 12 years from now.

Example 4: The population of the world in 2000 was 6.1 billion, and the estimated relative growth rate was 1.4% per year. If the population continues to grow at this rate, when will it reach 122 billion?

Example 5: The number of Bacteria in a Culture

A culture starts with 10,000 bacteria, and the number doubles every 40 minutes.

a. Find a function that models the number of bacteria at time $t$.
b. Find the number of bacteria after one hour.
c. After how many minutes will there be 50,000 bacteria?
Radioactive Decay: If $m_0$ is the initial mass of a radioactive substance with half-life $h$, then the mass remaining at time $t$ is modeled by the function $m(t) = m_0e^{-rt}$

Example 6: Radioactive Decay

Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

a. Find a function that models the amount of the sample remaining at time $t$.
b. Find the mass remaining after one year.
c. How long will it take for the sample to decay to a mass of 200 mg?
d. draw a graph of the sample mass as a function of time.

Logarithmic Scales

Chemists measured the acidity of a solution by giving its hydrogen ion concentration until Sorensen, in 1909, proposed a more convenient measure. He defined $pH = -\log[H^+]$ where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

If $[H^+] = 10^{-4}M$, then $pH = -\log(10^{-4}) = -(−4) = 4$

Solutions with a pH of 7 are defined as neutral, those with pH < 7 are acidic, and those with pH > 7 are basic. Note: when the pH increases by one unit, $[H^+]$ decreases by a factor of 10.

The Richter Scale: $M = \log \frac{I}{I_S}$

The Decibel Scale: $\beta = 10\log \frac{I}{I_o}$
The Richter Scale: \( M = \log \frac{I}{S} \)

The 1906 earthquake in San Francisco had an estimated magnitude of 8.3 on the Richter scale. In the same year, the strongest earthquake ever recorded occurred on the Colombia-Equador border, and was four times as intense. What was the magnitude on the Richter scale?

The 1989 Loma Prieta Earthquake that shook San Francisco had a magnitude of 7.1 on the Richter scale. How many times more intense was the 1906 earthquake?

The Decibel Scale \( \beta = 10 \log \frac{I}{I_o} \)

Given: \( I_o = 10^{-12} W/m^2 \) the threshold of hearing in watts per sq. meter

Find the decibel intensity level of a jet engine during takeoff if the intensity was measured at 100\( W/m^2 \).
Four most common mathematical models

1) Exponential growth \( y = ae^{bx} \)

2) Exponential decay \( y = ae^{-bx} \)

3) Logistics growth model \( y = \frac{a}{1+be^{-(x-c)/d}} \)

4) Logarithmic models \( y = a + b \ln x \)
   \( y = a + b \log x \)
Examples: The population \( P \) of a city is given by \( P = 240,360 e^{0.012t} \) where \( t = 0 \) represents 1990. According to this model, when will the population reach 275,000?

Solution: \( 275,000 = 240,360 e^{0.012t} \)

\[
\frac{275000}{240360} = e^{0.012t}
\]

\[
\ln \left( \frac{275,000}{240,360} \right) = \ln e^{0.012t}
\]

\[
\ln \frac{275,000}{240,360} = 0.012t
\]

\[
\ln \frac{275,000}{240,360} = t
\]

\[
\frac{250}{3} \ln \frac{275,000}{240,360} = t
\]

\( t \approx \)

Example 2: Find an exponential growth model whose graph passes through \((0, 4453)\) and \((7, 5024)\)

\[y = ae^{bx}\]

substitute for \((x, y) \rightarrow (0, 4453)\)

\[4453 = ae^{b \cdot 0}\]

\[4453 = ae^0\]

\[4453 = a\]

\[y = 4453e^{bx}\]

substitute for \((x, y) \rightarrow (7, 5024)\)

\[5024 = 4453e^{7b}\]

\[
\frac{5024}{4453} = e^{7b}
\]

\[
\ln \left( \frac{5024}{4453} \right) = 7b
\]

\[
\frac{1}{7} \ln \left( \frac{5024}{4453} \right) = b
\]

\( b \approx 0.01724\)

\[y \approx 4453e^{0.01724x}\]
To estimate the age of dead organic material use the following formula:

\[ R = \frac{1}{10^{12}} e^{-t/8223} \]

The ratio of carbon 14 to carbon 12 in a newly discovered fossil is \( R = \frac{1}{10^{13}} \)

Estimate the age of the fossil.

\[
\frac{1}{10^{13}} = \frac{1}{10^{12}} e^{-\frac{t}{8223}} \\
10^{12} \cdot \frac{1}{10^{13}} = 10^{12} \cdot \frac{1}{10^{12}} e^{-\frac{t}{8223}} \\
\frac{1}{10} = e^{-\frac{t}{8223}} \\
\ln .1 = -\frac{t}{8223} \\
-8223 \ln .1 = t \\
\]

\[ t \approx 18,934 \]

Logistics Growth Models

populations which initially have rapid growth, followed by a declining rate of growth.

On a college campus of 5000 students, one student returns from vacation with a contagious flu virus. The spread of the virus is modeled by:

\[ y = \frac{5000}{1 + 4999e^{-0.8t}}, \quad t \geq 0 \]

\( y \) is the total number of students infected after \( t \) days.

The college will cancel classes when 40% or more of the students are ill.

a) how many students are infected after 5 days?

b) after how many days will classes be cancelled?