Math150 Lecture Notes 7.5
Solving Trigonometric Equations

Given \( y = x^2 - 5x + 6 \) find x-intercepts

Solution: \( 0 = x^2 - 5x + 6 \)

\[
0 = (x - 2)(x - 3)
\]
\( x = 2, \quad x = 3 \)

Given \( y = \sin \pi x + \cos \pi x \)

Solution

\[
0 = \sin \pi x + \cos \pi x
\]

\[-\cos \pi x = \sin \pi x \]

\[
\begin{align*}
\frac{-\cos \pi x}{-\cos \pi x} &= \frac{\sin \pi x}{-\cos \pi x} \\
1 &= -\tan \pi x
\end{align*}
\]

\[-1 = \tan \pi x \]

\[
\arctan(-1) = \pi x
\]

\[
\arctan(-1) = \pi x
\]

\[
\frac{\pi}{\pi} = x
\]

\[
x = \frac{-1}{4}
\]

\[
x = \frac{-1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{11}{4}, \ldots
\]

Verify \( \frac{5\pi}{6} \) is a solution of \( \csc x - 2 = 0 \)

\[
\begin{align*}
\csc \frac{5\pi}{6} - 2 &= 0 \\
\sin \frac{5\pi}{6} &= \frac{1}{2}
\end{align*}
\]

\[
\csc \frac{5\pi}{6} = 2 \\
\sin 150^\circ = \frac{1}{2}
\]
Solving a Trigonometric Equation

\[ 2 \sin x - 1 = 0 \]

\[ 2 \sin x = 1 \]

\[ \sin x = \frac{1}{2} \quad \text{sin } \theta \text{ is positive in QI and QII} \]

\[ x = 30^\circ \quad \text{or} \quad x = \frac{\pi}{6} \quad \text{OR} \quad x = 150^\circ \quad \text{or} \quad x = \frac{5\pi}{6} \]

Solution: \( x = \frac{\pi}{6} + 2\pi n \) and \( x = \frac{5\pi}{6} + 2\pi n \)

since sine has a period of \( 2\pi \).

Solve: \( \sin x + \sqrt{2} = -\sin x \)

\[ \sqrt{2} = -2 \sin x \]

\[ -\frac{\sqrt{2}}{2} = \sin x \]

sine function is negative in QIII and QIV

reference angle is \( 45^\circ \) or \( \frac{\pi}{4} \)

\[ \pi + \frac{\pi}{4} = \frac{5\pi}{4} \]

\[ 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \]

period of sine function is \( 2\pi \)

\[ x = \frac{5\pi}{4} + 2\pi n \quad \text{and} \quad x = \frac{7\pi}{4} + 2\pi n \]

Solve: \( 3 \tan^2 x - 1 = 0 \)

\[ 3 \tan^2 x = 1 \]

\[ \tan^2 x = \frac{1}{3} \]

\[ \tan x = \pm \sqrt{\frac{1}{3}} \]
\[ \tan x = \pm \frac{1}{\sqrt{3}} \]

period of tangent function is \( \pi \)

find all solutions \([0, \pi) \rightarrow QI, QII\)

\[
\theta = \frac{\pi}{6}
\]

\[
\theta' = \pi - \frac{\pi}{6} = \frac{5\pi}{6}
\]

General solutions: \( \theta = \frac{\pi}{6} + \pi n \) and \( \theta = \frac{5\pi}{6} + \pi n \)

Solve: \( \cot x \cdot \cos^2 x = 2 \cot x \)

\[
\cot x \cdot \cos^2 x - 2 \cot x = 0
\]

\[
cot x (\cos^2 x - 2) = 0
\]

\[
cot x = 0 \quad \text{or} \quad \cos^2 x - 2 = 0
\]

\[
x = \frac{\pi}{2} \quad \cos^2 x = 2
\]

\[
\cos x = \pm \sqrt{2}
\]

\[
\cos x \approx \pm 1.414...
\]

\( \phi : \text{NO Solution} \)

Solution: \( x = \frac{\pi}{2} + \pi n \)

**Find all solutions of:** \( 2 \sin^2 x - \sin x - 1 = 0 \) from \([0, 2\pi)\)

\[
(2 \sin x + 1) (\sin x - 1) = 0
\]

\[
2 \sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0
\]

\[
2 \sin x = -1
\]

\[
\sin x = -\frac{1}{2} \quad \sin x = 1
\]
sine function is negative in QIII, and QIV.
reference angle is $\frac{\pi}{6}$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

Sometimes to solve an equation, we need to first rewrite with a single trigonometric function.

$$2 \sin^2 x + 3 \cos x - 3 = 0$$
$$2(1 - \cos^2 x) + 3 \cos x - 3 = 0 \quad \sin^2 x + \cos^2 x = 1$$
$$2 - 2 \cos^2 x + 3 \cos x - 3 = 0 \quad \sin^2 x = 1 - \cos^2 x$$
$$0 = 2 \cos^2 x - 3 \cos x + 1$$
$$0 = (2 \cos x - 1)(\cos x - 1)$$
$$2 \cos x - 1 = 0 \quad \cos x - 1 = 0$$
$$\cos x = \frac{1}{2} \quad \cos x = 1$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \quad x = 0$$

General solution $x = 0 + 2\pi n, \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

Find all solutions over the interval $[0, 2\pi)$

$$\cos x + 1 = \sin x$$

$$(\cos x + 1)^2 = \sin^2 x$$

$$\cos^2 x + 2 \cos x + 1 = 1 - \cos^2 x$$

$$2 \cos^2 x + 2 \cos x = 0$$

$$2 \cos x(\cos x + 1) = 0$$

$$\cos x = 0, \quad \cos x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi$$

Check for extraneous roots.
Solution: \( x = \frac{\pi}{2} \pi \)

Find all solutions of: \( 2 \cos 3t - 1 = 0 \)

Find all solutions of: \( 3 \tan \left( \frac{x}{2} \right) + 3 = 0 \)

Find all solutions of: \( \sec^2 x - 2 \tan x = 4 \)

\[
1 + \tan^2 x - 2 \tan x = 4
\]

\[
0 = \tan^2 x - 2 \tan x - 3
\]

\[
0 = (\tan x - 3)(\tan x + 1)
\]

\[
\tan x = 3 \quad \tan x = -1
\]

\[
\arctan 3 = x \quad x = -\frac{\pi}{4}
\]

\[
x = \arctan 3 + n\pi \quad x = -\frac{\pi}{4} + n\pi
\]