Ellipses

An ellipse is the set of all points in the plane the sum of whose distances from two fixed points is a constant.

If the origin is placed halfway between the foci, the equation is found using the distance formula.

\[ ||F_1P|| = \sqrt{(x+c)^2 + y^2} \quad ||F_2P|| = \sqrt{(x-c)^2 + y^2} \]

\[ \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = \text{constant} \]

\[ \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a \]

\[ \frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1 \]

Let \( b^2 = a^2 - c^2 \)

Then the equation of the ellipse with a Horizontal axis is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad a > b > 0 \)

Standard Form for the Equation of an Ellipse with Foci at \( (c,0) \) and \( (-c,0) \)

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where} \quad a > b > 0 \quad \text{and} \quad c^2 = a^2 - b^2 \]

The vertices are at \( (\pm a, 0) \), and the foci at \( (\pm c, 0) \). The foci are always located on the major axis.

The length of the major axis is \( 2a \); the length of the minor axis is \( 2b \).

If the ellipse has a vertical major axis: then the constant under \( y^2 \) is larger than the constant under \( x^2 \).
The Standard Form for the Equation of an Ellipse with Foci at (0,c) and (0,-c) is 
\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \text{ where } a > b > 0 \text{ and } c^2 = a^2 - b^2. \]

Sketch the graph of \(16x^2 + 9y^2 = 144\)

Find the equation of the ellipse whose vertices are at \((-4,0)\) and \((4,0)\) and whose foci are at \((-1,0)\) and \((1,0)\)

Definition of Eccentricity
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ with } a > b > 0, \text{ the eccentricity } e = \frac{c}{a} \]

The eccentricity of every ellipse satisfies \(0 < e < 1\), and \(c = \sqrt{a^2 - b^2}\)

If \(e\) is close to 1, the ellipse is elongated.

If \(e\) is close to 0, the ellipse is more circular.
Find the equation of the ellipse with foci at \((0, \pm 8)\) and eccentricity \(e = \frac{4}{5}\).

If the center of the ellipse is shifted from \((0,0)\) to \((h,k)\), its equation is:

- With a **horizontal** major axis:
  \[
  \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
  \]

- With a **vertical** major axis:
  \[
  \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
  \]

Given \( \frac{(x - 2)^2}{16} + \frac{(y + 5)^2}{9} = 1 \)

Find the vertices.

Graph: \(9x^2 + 4y^2 + 54x - 8y + 49 = 0\)

Sketch the graph of the following:

\(36x^2 + 4y^2 = 1\)