Test I

Instructions: Show all work in your bluebook. Calculators that do linear algebra or calculus are not allowed.

1. (10 pts.) Find both the parametric equation for the plane passing through the three points \( P(1,0,1), Q(1,2,2), R(0,1,1) \) and the area of the triangle \( \triangle PQR \).

2. (10 pts.) Let \( x = (3,2,-3,-1) \) and \( v = (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}) \). Find the component of \( x \) in the direction of \( v \) (i.e., the projection \( p \) of \( x \) on \( v \)), and the component \( q \) of \( x \) perpendicular to \( v \).

3. A linear system \( Ax = b \) has the augmented matrix \([A|b]\) given below.

\[
[A|b] = \begin{pmatrix}
1 & 1 & -3 & -2 & 2 \\
-1 & -1 & 4 & 3 & -1 \\
1 & 1 & -2 & -1 & 3
\end{pmatrix}
\]

(a) (10 pts.) The corresponding homogeneous system is \( Ax = 0 \). Is the system overdetermined or underdetermined? Does it have any non-trivial (i.e., non-zero) solutions? Are the columns of \( A \) linearly dependent or linearly independent?

(b) (15 pts.) Put \([A|b]\) in reduced row echelon form. Write the solution to \( Ax = b \) in parametric form.

4. (20 pts.) Use row reduction either to find \( C^{-1} \) or to show that it does not exist, given that the matrix \( C \) is

\[
C = \begin{pmatrix}
1 & -1 & 3 \\
2 & 3 & 4 \\
-1 & 3 & -4
\end{pmatrix}.
\]

5. (15 pts.) Evaluate \( \det(B) \), where \( B \) is given below, using any method. Using your answer, determine whether \( B \) is singular or invertible.

\[
B = \begin{pmatrix}
1 & 2 & 0 & 0 \\
-2 & -3 & 0 & 2 \\
0 & 1 & 2 & 0 \\
2 & 0 & -1 & 3
\end{pmatrix}.
\]
6. (10 pts.) Find $\det(FG)$, given that $F$ and $G$ are the matrices below.

$$F = \begin{pmatrix} 2 & 0 & 0 \\ -2 & -3 & 0 \\ 9 & 1 & 2 \end{pmatrix}, \quad G = \begin{pmatrix} -5 & 9 & 7 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$ 

7. (10 pts.) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear function for which $f([1 \ -1]^T) = [1 \ 2]^T$ and $f([1 \ 1]^T) = [-2 \ 3]^T$. Find the matrix $A$ that represents the function $f$. 
