1. Let \( V = \{ (x, y) | x, y \text{ are real numbers} \} \). Addition and scalar multiplication are defined on \( V \) as follows:
\[
(x, y) + (x', y') = (x+x', y+y') \quad \text{(usual definition)}
\]
\[
c \cdot (x, y) = (cx, cy)
\]
For each of the 10 axioms for a vector space check if it holds for \( V \). Show your work. (5 pts)

2. Which of the following subsets of \( \mathbb{R}^{3 \times 3} \) are subspaces. Justify all your answers (1 pt each)
(a) The set \( T \) of all matrices with integer entries
(b) The set \( T \) of all matrices with trace 0 (The trace of a matrix is the sum of its diagonal elements)
(c) The set \( D \) of all matrices with determinant 0
(d) The set \( S \) of all skew symmetric matrices (a matrix \( A \) is skew symmetric if \( A^T = -A \))
(e) The set \( I \) of all matrices \( X \) that commute with a fixed matrix \( B \) \( (XB = BX) \)

3. Let \( \overline{v}_1 = (1, 1, -1)^T \), \( \overline{v}_2 = (2, -1, 0)^T \), \( \overline{v}_3 = (2, 2, -2)^T \), \( \overline{v}_4 = (3, 1, 2)^T \)
(a) show that \( \overline{v}_1, \overline{v}_2, \overline{v}_3, \overline{v}_4 \) span \( \mathbb{R}^3 \). (3 pts)
(b) Is there any proper subset of these vectors that span \( \mathbb{R}^3 \)? Justify your answer. (2 pts)

4. a) Do the matrices \( A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \), \( B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \), \( C = \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix} \) span \( \mathbb{R}^{3 \times 3} \). Justify your answer. (3 pts)
b) Is the matrix \( \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) in the span of \( A, B, C \)? Justify your answer. (2 pts)