1. Pare down the set \{1 + x + x^2, x - 2x^2, 1 + 2x - x^2, 1 + 3x^2, 1 + x\} to a basis of \( \mathbb{P}_3 \). (3 pts)

2 a) Find a basis for the space of 2x2 symmetric matrices.
Prove that your answer is indeed a basis. (3 pts)
b) Find the dimension of the space of \( n \times n \) symmetric matrices.
Justify your answer. (1 pt)

3 a) Compute the wronskian of \{ \sin x, \cos x, \sin(x + \frac{\pi}{3}) \}. (1 pt)
Are these functions linearly dependent? Justify your answer. (1 pt)
b) Consider the function \( g_1 = x \), \( g_2 = x^{\frac{1}{3}} \) in \( C[-2,2] \).
Can we use the wronskian to decide if \( g_1 \) and \( g_2 \) are linearly dependent? Justify your answer.
c) Are \( g_1 \) and \( g_2 \) linearly dependent? Justify your answer. (1 pt)

4. Let \( U \) and \( V \) be two 2-dimensional subspaces of \( \mathbb{R}^3 \). Prove that \( U \cap V \neq \{ \mathbf{0} \} \). (2 pts)

5. Let \( A = \begin{pmatrix}
1 & 2 & -1 & 3 & 2 \\
-1 & 3 & 0 & 1 & 1 \\
1 & 2 & 1 & 4 & 2 \\
2 & -1 & 1 & 3 & 1
\end{pmatrix} \)

a) Find a basis for row space (A). Express every row of \( A \) as a linear combination of the basis. (2 pts)
b) Find a basis for column space (A). Express every column of \( A \) as a linear combination of the basis. (2 pts)
c) Find a basis for the null space of \( A \). Check that the Rank-Nullity Theorem is true for \( A \). (2 pts)