1. Let \( v \) denote a smooth vector field defined on a bounded, open subset \( \mathcal{R} \) of \( \mathbb{R}^3 \). Prove that
\[
\int_{\mathcal{R}} \nabla v \, dV = \int_{\partial \mathcal{R}} v \otimes n \, dA
\]
where \( n \) denotes the unit normal vector field to the boundary \( \partial \mathcal{R} \) of \( \mathcal{R} \).

2. Let \( u \) denote a smooth vector field on \( \mathbb{R}^3 \). Derive the following identities.
   
   (a) \( \text{Div}\{ (\nabla u)u \} = \nabla u \cdot \nabla u^T + u \cdot (\nabla \text{Div} u) \)
   
   (b) \( \nabla u \cdot \nabla u^T = \text{Div}\{ (\nabla u)u - (\text{Div} u)u \} + (\text{Div} u)^2 \).

3. Let \( E[\cdot] \) denote a constant, fourth-order elasticity tensor. Consider the Navier system of partial differential equations
\[
\ddot{u}(t, x) = \text{Div} E[\nabla u(t, x)].
\]

A **progressive wave** is a vector field of the form:
\[
u(t, x) := \phi(x \cdot m - ct)a
\]
where \( c \) is a constant scalar wave speed, and \( a \) and \( m \) are the constant vector wave amplitude and wave direction, respectively.

(a) Show that (2) satisfies (1) if and only if
\[
A(m)a = c^2a
\]
where \( A(m) \) denotes the **Acoustic Tensor** defined by the relation:
\[
A(m)a := E[a \otimes m]m.
\]

(b) Show that the Acoustic Tensor satisfies:
   
i. \( A(m)^T = A(m) \)
   
ii. \( a \cdot A(m)a = a \otimes m \cdot E[a \otimes m] \).