1. A company's profit for producing and selling $x$ units of a product is given by 
   \[ P(x) = -0.2x^2 + 50x - 2500. \]
   Use the marginal profit to approximate the change in profit if production increases from 150 to 151 units.

2. Shown is the graph of $f'(x)$, the derivative of $f(x)$.

   ![Graph of f'(x)]

   a) List the interval(s) on which $f$ is increasing.

   b) At what $x$-value(s) does $f$ have a local max?

   c) List the interval(s) on which $f$ is concave up.

   d) At what $x$-value(s) does $f$ have an inflection point?

3. Find the derivative with respect to $x$ of each function. **Do not simplify your answer.**

   a) $f(x) = (x - 2)^2 e^{4x}$

   b) $g(x) = \ln \left[ \frac{x^7 \sqrt{x^3 + 5}}{6x^2 + 4} \right]$
c) \[ h(x) = \frac{x^2}{e^x + 1} \]

d) \[ m(x) = 3e^{\sqrt{x^2 + 1}} + 25 \]

4. The demand quantity, \( x \) and price, \( p \), for a product are related by \( x^2 + 2p = 1200 \).

a) Find the elasticity function, \( E(p) \).

b) On what interval is demand inelastic?

5. The elasticity of demand for a product is given by \( E(p) = \frac{p^2}{18 - p^2} \).

a) At what price, \( p \), is revenue a maximum?

b) Find the approximate per-cent change in demand if the price increases from \$2.00\) to \$2.14.

c) If the price increases from \$2.00\) to \$2.14\), then demand will \( \text{increase} / \text{decrease} \) (circle one)

and revenue will \( \text{increase} / \text{decrease} \). (circle one)
6. The derivative of $f(x)$ is given by $f'(x) = \frac{x^2}{(x-1)^3}$. $f(x)$ is continuous for all $x$ except $x=1$, where $f$ has a vertical asymptote.

a) Determine the intervals on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

b) Find $f''(x)$ and simplify it.

c) Determine where $f$ is concave up / concave down. Find the x-value(s) of any and all inflection point(s).

7. Find the absolute maximum and absolute minimum of $f(x) = (x-2)^2(x+4)$ on the interval [-3, 5]. You must show your work.

8. A company's cost for producing $x$ units of a product is given by $C(x) = 0.5x^2 + 30x + 200$.

a) Find the marginal average cost function.

b) Find the quantity at which the average cost is a minimum.

c) Use the 2nd derivative test to show this x-value minimizes the average cost.

9. The current market price for a product is $12 and is decreasing by $0.50 per month. A store owner can currently sell 20 units per month and his customer demand is increasing by 3 per month. In how many months will the store owner's revenue be a maximum? Give an exact answer. Verify that your answer is the maximum.
10. A train uses $400 per hour for labor and other costs. In addition, at speed $v$ it uses \( \frac{v^2}{3} \) dollars per hour for fuel. What speed should it travel to minimize total cost. You may assume the train travels 100 miles. Hint: time = distance /speed Verify that your answer minimizes the cost by using the 2nd derivative test.

11. \( f(x) = e^x \) \( g(x) \) and \( g''(x) \) exists for all \( x \).
   a) Write the product rule as it applies to \( f(x) \).

b) Similarly write out \( f''(x) \).

c) Given that \( g(0) = 0 \), \( g'(0) = 0 \), and \( g''(0) = -2 \), what can concluded, if anything, about \( f \) at \( x = 0 \)?

d) Given that \( g(1) = -1 \), \( g'(1) = 1 \) and \( g''(1) = -1 \), what can be concluded, if anything, about \( f \) at \( x = 1 \)?

12. The graph of \( f(x) \) is shown.

List any/all \( x \)-values within the graph at which:

a) \( f \) has no derivative.

b) \( f'(x) = 0 \)

c) \( f \) has a local minimum.

d) \( f \) has a local maximum.
13. Use differentiation rules to find the derivative of each function below.

a) \((x^2 + 1)e^x\)

b) \(e^{(x^3 - 2x^2 + 4x + 5)}\)

c) \(\frac{\ln x}{x^4 + 1}\)

d) \(\ln \left[ \frac{(x^2 - 5)^3(5x + 7)^2}{2x + 3} \right] \) Use log rules!

14. A profit function is \(P(x) = -0.15x^2 + 90x - 540\) where \(x\) is the quantity produced and sold.

a) Find the marginal profit function.

b) Use the marginal profit function to approximate the change in profit if production increases from 200 to 201.

c) Find the \textbf{marginal average} profit function.

d) Find the quantity at which average profit is at a maximum.

15. For a certain product, the demand quantity as a function of price/unit is \(f(p) = \sqrt{600 - p^2}\). Find the elasticity function, \(E(p)\).

16. An elasticity function is \(E(p) = 0.01p^2\).

a) At what price is revenue at a maximum?

b) On what price range is demand inelastic? Is revenue increasing or decreasing on this price range?

c) Approximate the % decrease in demand if price increases from $8 to $9.
17. The derivative of \( f(x) \) is \( f'(x) = (x+1)^3(x-2)^2 \).
   
a) Make a sign chart for \( f'(x) \) and find the \( x \)-value(s) of any local max or min of \( f \).
   
b) Find \( f''(x) \), make a sign chart, and locate any inflection point(s) of \( f \).

18. The derivative of \( f(x) \) is \( f'(x) = \frac{x}{x^2 + 36} \).
   
a) Find \( f''(x) \) and simplify it.
   
b) Make a sign chart and locate any inflection point(s) of \( f \).

19. A company's profit for producing and selling \( x \) units of a product is given by \( P(x) = -0.3x^2 + 70x - 3000 \).
   
a) Use the marginal profit to approximate the change in profit if production increases from 120 to 121 units.
   
b) Use the marginal profit and \( P(120) = 1080 \) to approximate the profit if 121 units are produced.

20. Find the **absolute** max and **absolute** min of \( f(x) = x^3 - 3ax^2 \) on the interval \( [0, 4a] \).

21. Find the derivative with respect to \( x \) of each function. **Do not simplify your answer.**

   \[
   a) \quad f(x) = 4 \left(5^{x^3 + 2x}\right) + 32
   \]

   \[
   b) \quad g(x) = \ln \left[ \frac{(x^2 + 1)\sqrt{x^4 + 5}}{2x^3 + 6} \right]
   \]
22. The demand quantity for a product at price \( p > 10 \) is given by \( f(p) = pe^{-0.1p} \).

a) Find the elasticity function, \( E(p) \).

b) At what price, \( p \), is revenue at a maximum?

23. The elasticity of demand for a product at price $2.00 per unit is \( E(2) = 0.6 \).
   a) Find the approximate percent change in demand if the price increases from $2.00 to $2.10.
   b) Under this price increase, revenue will increase / decrease and demand will increase / decrease.

24. The first derivative of a function \( g(x) \) is given by \( g'(x) = (x+1)^3 (x-2)^2 \).
   a) Determine the intervals on which \( g \) is increasing / decreasing and locate any relative max / min of \( g \).
   b) Find the 2nd derivative of \( g(x) \) and factor it.
   c) Determine the intervals on which \( g \) is concave up/ concave down and list all inflection points of \( g \).

25. A company's cost for producing \( x \) units of a product is given by
   \( C(x) = 0.25x^2 + 60x + 144 \).

a) Find the \textbf{Marginal Average Cost} function.

b) Find the critical value of the average cost and use the 2nd derivative test to classify it as a max or min.

26. The current market price for a product is $15 and is decreasing by $0.50 per month. A store can currently sell 30 units per month and his customer demand is increasing by 2 units per month. In how many months will his monthly revenue be a maximum? Give an exact number of months, do not round. Show your answer gives a max using the 2nd derivative test.