Evaluate using the fundamental theorem of calculus.

1. \( \int_{1}^{3} e^{4x} \, dx \)  
2. \( \int_{0}^{2} \sqrt{x+1} \, dx \)  
3. \( \int_{1}^{2} \frac{t}{t^2 + 1} \, dt \)  
4. \( \int_{0}^{2} xe^{x^2} + (x+1)^2 \, dx \)  
5. \( \int_{0}^{a} \frac{5t + 10}{(t^2 + 2t + 5)^{1/3}} \, dt \)  
6. \( \int_{0}^{b} 7x\sqrt{x^2 + 1 + e^{3x}} \, dx \)  
7. Use symmetry to evaluate \( \int_{-2}^{2} \frac{t^3}{t^2 + 1} \, dt \).  
Use substitution to evaluate the same integral.

8. Find \( \frac{d}{dx} \left[ \int_{0}^{x} e^{t^2} \, dt \right] \) and \( \int_{0}^{x} \frac{d}{dx} (e^{x^2}) \, dx \).

9. Shown is the graph of \( F'(x) = f(x) \).
Find a) \( F(4) - F(0) \)  
b) \( F(2) - F(0) \)  
c) \( F(4) - F(2) \)
10. Shown is the graph of \( f(x) \).

Evaluate each definite integral.

\[ a) \int_{0}^{10} f(x) \, dx \quad b) \int_{3}^{8} f(x) \, dx \]

\[ c) \int_{0}^{4} (f(x) + 2x^2) \, dx \]

d) Find the average value of \( f(x) \) over the interval \([3, 10]\).

11. The total production cost in dollars per unit for a certain product is given by \( C(x) = -0.2x^2 + 30x + 1500 \). Find the average value of the cost function for the interval \([0, 200]\). Compare to the average cost per unit if 200 units are produced.

12. A marginal average cost function is given by

\[ MAC = 0.5 \cdot \frac{2050}{x^2} \text{ dollars/unit/unit.} \]

If the total cost for the first 200 units is $10000, find

a) the cost function, \( C(x) \).

b) the average value of the total cost over \([100, 250]\).
13. Find the area between the two functions.
   \( a) \ f(x) = x^2 + 5x \quad g(x) = 7x + 3 \)

   \( b) \ f(x) = x^3 \quad g(x) = 4x^2 \)

14. Find the area between the two functions.
   \( a) \ f(x) = x^3 + 5x^2 \quad g(x) = 8x^2 + 10x \)

   \( b) \ f(x) = xe^{x^2} \quad g(x) = ex \)

15. Find the area bounded by the graphs of:
   \( a) \ f(x) = e^x, \quad g(x) = x, \quad x = 0, \text{ and } x = 1 \)

   \( b) \ f(x) = x^2 + 5x, \quad g(x) = 7x + 3, \quad x = 0 \text{ and } x = 4 \)

   \( c) \ f(x) = e^x, \quad g(x) = x + 1, \quad x = -1 \text{ and } x = 2 \)

16. Find the consumers' surplus at price level $30 for the demand function
   \( D(x) = 50 - e^{0.1x} \) dollars per unit.

17. Find the consumers' surplus and the producers' surplus at equilibrium for the given demand and supply equations.
   \( a) \ D(x) = 87 - 0.2x^2 \quad S(x) = 15 + 0.3x^2 \)

   \( b) \ D(x) = 80 - \sqrt{0.2x + 4} \quad S(x) = 20 + 3\sqrt{0.1x} \)