Quiz 3A (Answers)

1. Consider the ODE

\[(1)\quad y'' + 7y' + 6y = 0.\]

You may assume without proof that \(y_1(x) = e^{-x}\) and \(y_2(x) = e^{-6x}\) are solutions to (1) on the interval \((-\infty, \infty)\).

(a) Verify that all solutions of (1) are given by \(y = c_1e^{-x} + c_2e^{-6x}\) where \(c_1\) and \(c_2\) are arbitrary constants.

(b) Find the solution of (1) satisfying the initial conditions \(y(0) = 3\), \(y'(0) = -8\).

**Answer.** In (a), one needs to check that \(y_1\) and \(y_2\) are a fundamental set of solutions to (1). This can be done by calculating the Wronskian:

\[
W(y_1, y_2)(x) = \begin{vmatrix} e^{-x} & e^{-6x} \\ -e^{-x} & -6e^{-6x} \end{vmatrix} = -5e^{-7x} \neq 0.
\]

Alternatively one can observe that \(y_1(x)/y_2(x) = e^{5x} \neq \text{const.}\)

In (b), the initial conditions imply that

\[
\begin{align*}
  c_1 + c_2 &= 3 \\
  -c_1 - 6c_2 &= -8,
\end{align*}
\]

which gives \(y(x) = 2e^{-x} + e^{-6x}\).