Quiz 3B (Answers)

1. Consider the ODE

\[ y'' + 4y' + 3y = 0. \]  

You may assume without proof that \( y_1(x) = e^{-x} \) and \( y_2(x) = e^{-3x} \) are solutions to (1) on the interval \( (-\infty, \infty) \).

(a) Verify that all solutions of (1) are given by \( y = c_1 e^{-x} + c_2 e^{-3x} \) where \( c_1 \) and \( c_2 \) are arbitrary constants.

(b) Find the solution of (1) satisfying the initial conditions \( y(0) = 0, y'(0) = 1 \).

Answer. In (a), one needs to check that \( y_1 \) and \( y_2 \) are a fundamental set of solutions to (1). This can be done by calculating the Wronskian:

\[ W(y_1, y_2)(x) = \begin{vmatrix} e^{-x} & e^{-3x} \\ -e^{-x} & -3e^{-3x} \end{vmatrix} = -2e^{-4x} \neq 0. \]

Alternatively one can observe that \( y_1(x)/y_2(x) = e^{2x} \neq \text{const.} \)

In (b), the initial conditions imply that

\[ c_1 + c_2 = 0 \]
\[ -c_1 - 3c_2 = 1, \]

which gives \( y(x) = (1/2)e^{-x} - (1/2)e^{-2x}. \)