Quiz 5B (Answers)

1. Given that \( y_1(x) = x \) is a solution of

\[
x^2 y'' - 3xy' + 3y = 0, \quad x > 0,
\]

find a second independent solution by deriving it using reduction of order. Do not plug into the formula.

**Answer.** Guess \( y(x) = v(x)y_1(x) = xv(x) \). Substitute \( xv(x) \) into (1) and simplify, obtaining \( x^3v''(x) - x^2v'(x) = 0, x > 0 \). Solving, one obtains \( v(x) = c_1 x^2 + c_2 \) and \( y_2(x) = c_1 x^3 + c_2 x \). \( y_2(x) \) will be a second independent solution for any nonzero value of \( c_1 \); in particular \( y_2(x) = x^3 \) suffices.

2. Consider the differential equation

\[
y'' + 4y = x + \cos(2x) \tag{2}
\]

(a) Find the general solution \( y_h \) to the associated homogeneous problem in (2).

(b) Find a particular solution \( y_p \) to (2).

(c) Find the general solution to (2).

**Answer.** In part (a), the auxiliary equation is \( r^2 + 4 = 0 \). The roots are \( r = \pm 2i \), so \( y_h = c_1 \cos(2x) + c_2 \sin(2x) \).

In part (b), one applies the method of undetermined coefficients, guessing that \( y_p = Ax + B + Cx \cos(2x) + Dx \sin(2x) \). Substituting and solving, one obtains \( y_p = (x/4) + (1/4)x \sin(2x) \).

In part (c),

\[
y = y_p + y_h = c_1 \cos(2x) + c_2 \sin(2x) + (x/4) + (1/4)x \sin(2x) \]