Quiz 13A (Answers)

1. Consider the initial value problem \( y'' + 6y' = 1, \ y(0) = 0, \ y'(0) = -1. \)

(a) Compute the Laplace transform of the given initial value problem, and solve for \( Y(s) \), the Laplace transform of the solution \( y(t) \).

(b) Find the partial fraction decomposition for \( Y(s) \).

(c) Solve for \( y(t) \) using the inverse Laplace transform of \( Y(s) \).

Answer. In (a), \( s^2Y(s) - sy(0) - y'(0) + 6sY(s) - 6y(0) = \frac{1}{s} \), so \( Y(s) = -\frac{s - 1}{s^2(s + 6)} \).

In (b), apply the method of partial fractions to compute \( Y(s) = \frac{1}{6s^2} - \frac{7}{36s} + \frac{7}{36(s + 6)} \).

Using the partial fraction decomposition in (b), one computes \( y(t) = \mathcal{L}^{-1}\{Y\}(t) = \frac{t}{6} - \frac{7}{36} + \frac{7}{36} e^{-6t} \).