Assignment 2–Solutions to Selected Problems

1. Problems 1 and 2, p. 138 in the text.
   **Answer.** Let $\phi, \psi : I \to I$ be defined as follows:
   \[
   \phi(t) = \begin{cases} 
   2t & \text{if } 0 \leq t < 1/2, \\
   1 & \text{if } 1/2 \leq t \leq 1
   \end{cases}
   \quad \text{and} \quad
   \psi(t) = \begin{cases} 
   0 & \text{if } 0 \leq t < 1/2, \\
   2t - 1 & \text{if } 1/2 \leq t \leq 1
   \end{cases}
   \]

   Obviously both $\phi$ and $\psi$ are homotopic to the identity rel $\partial I$. One notes that $f(0) = f(1) = g(0) = g(1) = e$ and that $f(t) = f(t) \bullet g(0) = f(t) \bullet g(1) = g(0) \bullet f(t) = g(1) \bullet f(t)$. Pointwise $f \ast g(t) = (f \circ \phi(t)) \bullet (g \circ \psi(t))$. It follows that $f \ast g \simeq f \bullet g$ rel $\ast$. This solves problem 1.

   For problem 2, observe that $g \ast f(t) = (f \circ \psi(t)) \bullet (g \circ \phi(t))$.

2. Problem 3, p. 138 in the text.
   **Answer.** Sketch. By Smooth Approximation and Sard’s Theorem, without loss of generality, we may assume that a loop is smooth and misses the base point $\ast$. Now the loop may be projected radially from $\ast$ to the “boundary” of the defining rectangle with identifications. One must adjust this argument slightly for the pointed homotopy.