Assignment 3–Solutions to Selected Problems

1. Problem 2, p. 143 in the text.
   **Answer.** Sketch. Let \( p : S^2 \to \mathbb{P}^2 \) be the antipodal identification, double covering map. Fixing a base point \( y_0 \in \mathbb{P}^2 \), let \( p^{-1}(y_0) = \{x_0, -x_0\} \) with \( x_0 \) the base point in \( S^2 \). By III.3.8, any \([f] \in \pi_1(\mathbb{P}^2)\) lifts to a path \( \hat{f} \) whose endpoint \( \hat{f}(1) \in \{x_0, -x_0\} \) depends only on the class \([f]\). In \( S^2 \), \( \hat{f} \) is null homotopic rel \( \partial I \) if \( \hat{f}(1) = x_0 \), and homotopic rel \( \partial I \) to a semicircle from \( x_0 \) to \( -x_0 \) if \( \hat{f}(1) = x_1 \). In the former case we must have \([f] = 1\), while in the latter case \([f] \neq 1\) by III.3.7.

2. Problem 3, p. 143 in the text.
   **Answer.** By Theorem III.2.6, we have \( \pi_1(\mathbb{T}^n) \) is the \( n \)-fold product of \( \pi_1(S^1) \), or \( \mathbb{Z} \times \cdots \times \mathbb{Z} \) (\( n \) factors).