Assignment 4–Solutions to Selected Problems

1. Problem 1, p. 145 in the text.
   **Answer.** Let \( f : S^n \to P^m \) represent a homotopy class in \( \pi_n(P^m) \). If \( p : S^m \to P^m \) is the canonical antipodal covering map, then \( f \) lifts to \( g : S^n \to S^m \) such that \( p \circ g = f \). But \( g \) is homotopic to a constant since \( 1 < n < m \), and \( f \) must be homotopically trivial.

2. Problem 2, p. 146 in the text.
   **Answer.** Let \( f : P^2 \to S^1 \). By Exercise III.3.2, \( \pi_1(P^2) = Z_2 \). So \( f_\#(\pi_1(P^2)) = \{1\} \) since \( \pi_1(S^1) = Z \). Thus \( f \) lifts to \( g : P^2 \to R \) such that \( f = p \circ g \), where \( p : R \to S^1 \) is the standard exponential covering map. But \( g \) is homotopically trivial since \( R \) is contractible, hence so is \( f \).