Week in Review #1

Section L.1: Introduction to Logic

- A **statement** is a declarative sentence that can be evaluated as either true or false (but not both).
- **Connectives**
  - conjunction (and), denoted $p \land q$
  - disjunction (inclusive or), denoted $p \lor q$
  - negation (not), denoted $\neg p$

1. Which of the following are statements.
   
   (a) A&M is the friendliest college in the world.
   
   (b) A&M’s Miss Reveille is a German Shepard.
   
   (c) There are 30 tennis courts on A&M’s campus.

2. Use the statements $b$, $s$, and $n$ for the following. compound statements in words.
   
   $b$: The car is blue. $s$: The car is a saturn. $n$: The car is new.
   
   (a) Express the compound statements in words.
       
       i. $n \land \neg b$
       
       ii. $s \lor b$
   
   (b) Give the symbolic expression for these statements.
       
       i. The new saturn was not blue.
       
       ii. The saturn was blue or it was not new.

Section L.2: Truth Tables

- **Definitions**
  - Exclusive Disjunction (exclusive or), denoted $\lor$
  - A **tautology** is a compound statement that is always true.
  - A compound statement that is always false is called a **contradiction**.

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3. Construct the following truth tables.

   (a) $\neg p \lor (p \land q)$
   
   (b) $p \land (\neg q \lor r)$

4. If the truth value of $p, q$ and $r$ is true and the truth value of $s$ is false, what is the truth value of these compound statements.

   (a) $(s \lor \neg r) \land q$
   
   (b) $(\neg q \land r) \lor (\neg s \land p)$
   
   (c) $p \lor \left[ \left( \neg r \land s \right) \lor \neg (q \land \neg p) \land r \right]$
Section 1.1: Set and Set Operations.

- A set is a well defined collection of objects
- Roster notation: \( A = \{1, 2, 3\} \)
- Set builder notation: \( B = \{x \mid x \text{ is a positive integer}\} \)

Definitions:
- \(x\) is an element of set \( A\), \( x \in A\), if \( x\) is an object in \( A\).
- Set \( A\) and \( B\) are equal if they have exactly the same elements.
- \( A\) is a subset of \( B\), \( A \subseteq B\), if every element in \( A\) is also an element of \( B\).
- \( A\) is a proper subset, \( A \subset B\), if \( A\) is a subset of \( B\) but is not equal to \( B\).
- The empty set, \( \emptyset = \{\}\), is a set that contains no elements.
- The universal set, \( U\), is the set that contains all of the elements possible in a problem.

- Set \( A\) and \( B\) are disjoint provided that \( A \cap B = \emptyset\).

Set operations:
- Union, \( A \cup B\)
- Intersection, \( A \cap B\)
- Complement, \( A^C\)

5. Write the set \( \{x \mid x\) is a letter in the word ENCYCLOPEDIA\} in roster notation.

6. \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\), \( A = \{0, 3, 6, 9\}\), \( B = \{0, 2, 4, 6, 8\}\), and \( C = \{1, 3, 5, 7, 9\}\).

Find the following.
- (a) \( n(A) = \)
- (b) \( A \cup B\)
- (c) \( A \cap C^C\)
- (d) \( A \cap B \cap C\)
- (e) \( (A \cap C)^C \cap B\)
- (f) How many subsets does \( B\) have?
- (g) How many proper subsets does \( B\) have?
- (h) Are \( A\) and \( B\) disjoint?
- (i) Are \( B\) and \( C\) disjoint?
- (j) Give two disjoint proper subsets of \( B\).

7. Shade the regions of a Venn Diagram that represent the following.
- (a) \( A \cup B \cup C\)
- (b) \((A^C \cap B) \cup C\)

8. Indicate the regions of the Venn Diagram that correspond to these set operations.

   - (a) \((B \cup C)^C\)
   - (b) \((A \cap C)^C \cap B\)

9. \( U\) = the set of A&M students.
- \( M = \{x \in U \mid x\) is male\}\)
- \( F = \{x \in U \mid x\) is female\}\)
- \( D = \{x \in U \mid x\) drinks Dr. Pepper\}\)
- \( S = \{x \in U \mid x\) drinks Sprite\}\)
- \( C = \{x \in U \mid x\) drinks coffee\}\)

(a) Describe each of the given sets in words.
- i. \( S \cup C^C\)
- ii. \( M \cap (D \cup S)\)

(b) Write the set (use set notation) that represents each of the given statements.
- i. The female students at A&M that drink sprite but do not drink coffee.
- ii. The students at A&M that drink coffee or do not drink Dr. Pepper.