Week in Review #1

Section L.1: Introduction to Logic

- A statement is a declarative sentence that can be evaluated as either true or false (but not both).
- Connectives
  - conjunction (and), denoted \( p \land q \)
  - disjunction (inclusive or), denoted \( p \lor q \)
  - negation (not), denoted \( \neg p \)

1. Which of the following are statements.
   
   (a) A&M is the friendliest college in the world.
   
   (b) A&M’s Miss Reveille is a German Shepard.
   
   (c) There are 30 tennis courts on A&M’s campus.

2. Use the statements \( b \), \( s \), and \( n \) for the following. compound statements in words.

   \( b \): The car is blue.
   
   \( s \): The car is a saturn.
   
   \( n \): The car is new.

   (a) Express the compound statements in words.

   i. \( n \land \neg b \)
   
   ii. \( s \lor b \)

   (b) Give the symbolic expression for these statements.

   i. The new saturn was not blue.

   ii. The saturn was blue or it was not new.

Section L.2: Truth Tables

- Definitions
  - Exclusive Disjunction (exclusive or), denoted \( \lor \)
  - A tautology is a compound statement that is always true.
  - A compound statement that is always false is called a contradiction.

\[
\begin{array}{c|c|c} 
\text{p} & \text{q} & \text{p} \land \text{q} \\
\hline 
\text{T} & \text{T} & \text{T} \\
\text{T} & \text{F} & \text{F} \\
\text{F} & \text{T} & \text{F} \\
\text{F} & \text{F} & \text{F} \\
\end{array}
\quad
\begin{array}{c|c|c} 
\text{p} & \text{q} & \text{p} \lor \text{q} \\
\hline 
\text{T} & \text{T} & \text{T} \\
\text{T} & \text{F} & \text{T} \\
\text{F} & \text{T} & \text{T} \\
\text{F} & \text{F} & \text{F} \\
\end{array}
\quad
\begin{array}{c|c|c} 
\text{p} & \text{q} & \text{p} \lor \text{q} \\
\hline 
\text{T} & \text{T} & \text{F} \\
\text{T} & \text{F} & \text{T} \\
\text{F} & \text{T} & \text{T} \\
\text{F} & \text{F} & \text{F} \\
\end{array}
\]

3. Construct the following truth tables.

(a) \( \neg p \lor (p \land q) \)
(b) \( p \land (\sim q \lor r) \)

4. If the truth value of \( p, q \) and \( r \) is true and the truth value of \( s \) is false, what is the truth value of these compound statements.

(a) \( (s \lor \sim r) \land q \)

(b) \( (\sim q \land r) \lor (\sim s \land p) \)

(c) \( p \lor \left[ (\sim r \land s) \lor (\sim (q \land \sim p) \lor r) \right] \)

Section 1.1: Set and Set Operations.

- a set is a well defined collection of objects
- roster notation: \( A = \{1, 2, 3\} \)
- set builder notation: \( B = \{x \mid x \text{ is a positive integer} \} \)
- Definitions:
  - \( x \) is an \textbf{element of} set \( A \), \( x \in A \), if \( x \) is an object in \( A \).
  - set \( A \) and \( B \) are \textbf{equal} if they have exactly the same elements.
  - \( A \) is a \textbf{subset} of \( B \), \( A \subseteq B \), if every element in \( A \) is also an element of \( B \)
  - \( A \) is a \textbf{proper subset}, \( A \subset B \), if \( A \) is a subset of \( B \) but is not equal to \( B \).
  - The \textbf{empty set}, \( \phi = \{\} \), is a set that contains no elements
  - The \textbf{universal set}, \( U \), is the set that contains all of the elements possible in a problem.
- Set \( A \) and \( B \) are \textbf{disjoint} provided that \( A \cap B = \phi \)
- Set operations:
  - Union, \( A \cup B \)
  - Intersection, \( A \cap B \)
  - Compliment, \( A^C \)
5. Write the set \{ x \mid x \text{ is a letter in the word ENCYCLOPEDIA} \} in roster notation.

6. \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \ A = \{0, 3, 6, 9\}, \ B = \{0, 2, 4, 6, 8\}, \) and \( C = \{1, 3, 5, 7, 9\} \)

Find the following.

(a) \( n(A) = \)

(b) \( A \cup B \)

(c) \( A \cap C^C = \)

(d) \( A \cap B \cap C = \)

(e) \( (A \cap C)^C \cap B = \)

(f) How many subsets does \( B \) have?

(g) How many proper subsets does \( B \) have?

(h) Are \( A \) and \( B \) disjoint?

(i) Are \( B \) and \( C \) disjoint?

(j) Give two disjoint proper subsets of \( B \).
7. Shade the regions of a Venn Diagram that represent the following.
   (a) \(A \cup B \cup C\)
   (b) \((A^c \cap B) \cup C\)

8. Indicate the regions of the Venn Diagram that correspond to these set operations.
   (a) \((B \cup C)^c\)
   (b) \((A \cap C)^c \cap B\)

9. \(U =\) the set of A\&M students.
   
   \[M = \{ x \in U | x \text{ is male}\}\]
   \[F = \{ x \in U | x \text{ is female}\}\]
   \[D = \{ x \in U | x \text{ drinks Dr. Pepper}\}\]
   \[S = \{ x \in U | x \text{ drinks Sprite}\}\]
   \[C = \{ x \in U | x \text{ drinks coffee}\}\]

   (a) Describe each of the given sets in words.
      i. \(S \cup C^c\)
      ii. \(M \cap (D \cup S)\)

   (b) Write the set (use set notation) that represents each of the given statements.
      i. The female students at A\&M that drink sprite but do not drink coffee.
      ii. The students at A\&M that drink coffee or do not drink Dr. Pepper.