Chapter 6 - Sets and Counting
6.1 - Sets and Set Operations

Set Terminology and Notation

Set: a well-defined collection of objects
Elements: objects in a set

Notation

Roster Notation: elements are all listed (like names on a roster)
Ex: $F = \{1, 2, 3, 4\}$
   $1 \in F$ ("1 is an element of $F$")
   $0 \notin F$ ("0 is not an element of $F$")

Ex: $H = \{0, 1, \ldots, 100\}$

Set-Builder Notation: a description of the elements in the set is given
- you decide if an object fits the description or not
Ex: $G = \{x | x \text{ is a student in one of Kathryn’s MATH 141 classes}\}$
Two sets are **equal** if and only if they have exactly the same elements. (Order doesn’t matter.)

**Ex:** \(A = \{a, b, c, d\}\) \(B = \{a, c, d\}\) \(C = \{d, c, a, b\}\)
\(R = \{x | x \text{ is a number divisible by } 2\}\) \(S = \{0, 2, 4, 6\}\)

Are any of these sets equal?

If every element of a set \(E\) is also an element of a set \(F\), then \(E\) is a **subset** of \(F\), denoted \(E \subseteq F\).

**Ex:** \(A = \{a, b, c, d\}\) \(B = \{a, c, d\}\) \(C = \{d, c, a, b\}\)
\(R = \{x | x \text{ is a number divisible by } 2\}\) \(S = \{0, 2, 4, 6\}\)

Are any of these sets subsets of one another?

Note: \(E\) is a **proper subset** of \(F\), denoted \(E \subset F\), if

1. \(E \subseteq F\) (All elements in \(E\) are also elements in \(F\).)
2. \(F\) is the larger set \((E \neq F)\)

**Ex:** \(A = \{a, b, c, d\}\) \(B = \{a, c, d\}\) \(C = \{d, c, a, b\}\)
\(R = \{x | x \text{ is a number divisible by } 2\}\) \(S = \{0, 2, 4, 6\}\)

Are any of these sets proper subsets of one another?

The set that contains no elements is called the **empty set**, denoted by either \(\emptyset\) or \(\{}\).

**The empty set is a subset of every set!**
Ex: \( A = \{-1, 2\} \quad B = \{e, f, g\} \)

List all of the subsets of \( A \).

List all of the subsets of \( B \).

In general, if a set has \( n \) elements then it will have \( 2^n \) subsets.

Note: 0 = the number zero
\[ \{0\} = \text{the set with one element, zero} \]
\[ \emptyset = \{\} = \text{the empty set} \]
\[ \{\emptyset\} = \text{the set with one element, the symbol for the empty set} \]
A **universal set**, $U$, is the set of all elements of interest in a particular problem.

We’ll use **Venn diagrams** to visually represent sets.

**Ex:** Let $U = \{x \mid x$ is a person living in College Station$\}$

$A = \{x \mid x$ is a person living in College Station and is enrolled at TAMU$\}$

$B = \{x \mid x$ is a person living in College Station and is enrolled in MATH 141 at TAMU$\}$

Draw a Venn diagram representing the relationship of these sets.

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**Ex:** Write down the following sets, using the given Venn diagram:

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A =

B =

U =
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Set Operations

Complement of a Set: $A^C = \text{the set of all elements in a universal set, } U,\text{ that are NOT in set } A$

Set Intersection: $A \cap B = \text{the set of all elements that belong to BOTH set } A \text{ AND set } B$
(The elements $A$ and $B$ have in common.)

If sets $A$ and $B$ have no elements in common then $A \cap B = \emptyset$ and $A$ and $B$ are said to be disjoint sets.
Set Union: \( A \cup B = \) the set of all elements that belong to EITHER set \( A \) OR set \( B \) OR BOTH SETS

**Pg. 340: Boxes give rules for set operations that you should look over to make sure they make sense.**

DeMorgan’s Laws

1. \((A \cup B)^C = A^C \cap B^C\)
2. \((A \cap B)^C = A^C \cup B^C\)
Ex: Given $U = \{1, 2, \ldots, 7\}$  $A = \{1, 2, 5, 7\}$  $B = \{3, 4, 5\}$  $C = \{1, 7\}$, find the following:

(a) $B^C$

(b) $(A \cup C) \cap B$

(c) $C^C \cap (A \cup B)^C$
Ex: Let $U = \{ x \mid x \text{ is a TAMU student} \}$

$M = \{ x \in U \mid x \text{ is a male} \}$

$B = \{ x \in U \mid x \text{ is a business major} \}$

$C = \{ x \in U \mid x \text{ is a member of the corps} \}$

Shade an appropriate Venn diagram to illustrate the following sets and then describe the sets in words.

(a) $(M \cup C)^C$

(b) $(M^C \cap C) \cup B$
Ex: Shade an appropriate Venn diagram illustrating the following sets:

(a) \( A \cap B \cap C \)

(b) \( (A \cup B^C) \cap C^C \)

(c) \( A^C \cap B \cap C^C \)