6.4 - Permutations and Combinations

Permutations

Arranging objects in a DEFINITE ORDER!

Ex: Find the number of permutations of the set \( A = \{1, 2, 3\} \) and list them.

Ex: Find the number of ways a group of five people can arrange themselves in a line.

Notice that when we take \( n \) distinct objects, \( n \) at a time, we get

\[ n(n - 1)(n - 2) \cdots (1) \]

ways of permuting the objects.

Notation

\( n \)-factorial \( = n! = n(n - 1)(n - 2) \cdots (1) \)

Ex: Evaluate

(a) 5!

(b) 0!

Calculator: # MATH → PRB ↓ 4!: ENTER
**Permutations of** \(n\) **DISTINCT Objects**

The number of permutations of \(n\) distinct objects, taken \(r\) at a time \((r \leq n)\), is given by

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

**Calculator:** \(n\) MATH \(\rightarrow\) PRB \(\downarrow\) 2:nPr \(r\) ENTER

**Ex:** How many three digit numbers can be formed from the digits 0-5 if no repetitions are allowed?

**Ex:** How many ways can a President, Vice President, and Secretary be chosen from a group of 50 people?

**Ex:** At the Miss America pageant, five girls from the 51 states (50 states, plus one from Washington D.C.) make it to the “top 5”, where each one wins a different amount of scholarship money. At the start of the pageant, how many different “top 5” arrangements are possible at the end of the pageant?
**Permutations of n Objects NOT ALL DISTINCT**

Given \( n_1 + n_2 + \cdots + n_r = n \) objects where \( n_1 \) objects are alike, \( n_2 \) objects are alike, etc. Then, the number of permutations of these \( n \) objects taken \( n \) (all) at a time is given by

\[
\frac{n!}{(n_1!)(n_2!)(n_r!)}
\]

**Ex:** Find the number of “distinguishable” ways the letters in the word MISSISSIPPI can be arranged.

**Ex:** On a planet, there are 12 martians whose genetics make them all look alike. Three are wearing identical blue outfits, 5 are wearing identical red outfits, 1 is wearing a green outfit, and the rest are wearing identical white outfits. If the martians stand in one row for a picture, how many “distinct” pictures would they be able to take?

**Combinations**

Arranging objects where ORDER DOES NOT MATTER!

The number of combinations of \( n \) objects, taken \( r \) at a time \((r \leq n)\), is given by

\[
C(n, r) = \frac{n!}{r!(n-r)!}
\]

**Calculator:** \( n \) MATH \( \rightarrow \) PRB \( \downarrow \) 3:nCr \( r \) ENTER

**Ex:** How many ways can a three member committee be formed from a group of 50 people?
Ex: How many five card poker hands can be dealt from a standard 52 card deck
(a) if you are counting all possible hands (no restrictions are placed)?

(b) that have five hearts?

(c) that have three 4’s and two Aces?

Ex: In a club with 7 male and 8 female members, how many four person committees can be made that have
(a) three males and one female?

(b) exactly two females?

(c) at least two females?
Ex: From among 80 light bulbs, 10 are defective. How many samples of 8, have
(a) exactly 1 defective light bulb?

(b) at least 1 defective light bulb?