1.5 - Rational, Radical, and Power Functions

Rational Functions

Def: A rational function has the form

\[ y = \frac{f(x)}{g(x)} \]

where \( f(x) \) and \( g(x) \) are polynomials and \( g(x) \neq 0 \).

Asymptotes

- Finding Vertical Asymptotes:

For the rational function \( h(x) = \frac{f(x)}{g(x)} \), if there is a value \( c \) that makes the denominator zero \( (g(c) = 0) \) and the numerator NOT zero \( (f(c) \neq 0) \), then \( x = c \) is a vertical asymptote (VA).

** NOTE: If there is a value \( c \) which makes BOTH the numerator and denominator of the rational function \( h(x) = \frac{f(x)}{g(x)} \) zero, then there is a “hole” in the graph of \( h(x) \) when \( x = c \).

Ex: Identify any holes or VA which occur in the graphs of the following functions:

(a) \( f(x) = \frac{x^2 - x - 2}{x^2 - 9} \)

(b) \( g(x) = \frac{2x}{x - 4} \)

(c) \( h(x) = \frac{x^2 + 6x + 8}{x^2 - x - 6} \)
**Finding Horizontal Asymptotes:**

**Horizontal asymptotes (HA)** describe the end behavior of a function **

For the rational function \( h(x) = \frac{f(x)}{g(x)} \):

- If the degree of \( f(x) \) is greater than the degree of \( g(x) \), then there is NO HA.
- If the degree of \( g(x) \) is greater than the degree of \( f(x) \), then there is a HA of \( y = 0 \).
- If the degrees of \( f(x) \) and \( g(x) \) are equal, then there is a HA of \( y = \frac{a}{b} \), where \( a \) and \( b \) are the leading coefficients of \( f(x) \) and \( g(x) \), respectively.

**Ex:** Find the HA of the following functions:

(a) \( m(x) = \frac{4x^3 + x^2 + 1}{2x^2 + 5x^4} \)

(b) \( n(x) = \frac{-2x^2 + 4}{x + 5} \)

(c) \( r(x) = \frac{3x^8 + 4x^6 - 3x^3 + 1}{2 + 5x^2 - 7x^8} \)

**Ex:** Determine the \( x \) and \( y \)-intercepts and all asymptotes of the following function and then make a sketch of the function.

\[
f(x) = \frac{(2x + 1)(x - 4)}{(x + 3)(x - 5)}
\]
Radical/Rational Exponent Functions

** RECALL: For real number $a$ and integers $m$ and $n$, the following are true:

- $a^0 = 1, a \neq 0$
- $a^{-n} = \frac{1}{a^n}, a \neq 0$
- $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

A radical/rational exponent function is of the form

$$f(x) = b\sqrt[n]{g(x)^a} = [g(x)]^{a/b}$$

Ex: Rewrite $f(x) = \sqrt[3]{(2x - 3)^3}$ as a function with rational exponents and state its domain.

Ex: Rewrite $g(x) = 2(x - 4)^{-1/3}$ as a radical function and state its domain.

Power Functions

Def: A function of the form

$$f(x) = a \cdot x^b$$

is called a power function where $a$ and $b$ are real numbers.

For $a > 0$:

| $0 < b < 1$ | $b > 1$ | $b < 0$ |
Ex: The number of electronics stores in Katbo can be modeled by the function

\[ f(x) = 19296x^{-0.029} \quad 1 \leq x \leq 20 \]

where \( x \) represents the number of years since 1980.

(a) Evaluate \( f(5) \) and interpret.

(b) Find the AROC in the number of electronic stores in Katbo for \( x = 5 \) and \( \Delta x = 10 \) and interpret.