2.3 - Instantaneous Rate of Change and the Derivative

**Ex:** Given \( f(x) = x^2 \), find the AROC of \( f(x) \) on the following intervals.

(a) \([1,2]\)

(b) \([1,1.5]\)

(c) \([1,1.2]\)

(d) \([1,1.1]\)

(e) Estimate the *instantaneous rate of change* of \( f(x) \) at \( x = 1 \).

**Ex:** Between which consecutive pairs of points is the AROC positive? negative? zero?

**Ex:** Using the figure in the previous example, determine whether the slope of the *tangent* line is positive, negative, or zero.
Instantaneous Rate of Change (IROC) / Slope of a Tangent Line / Derivative

**Def**: The IROC of a function, \( f \), at \( x \) is equal to the slope of the line tangent to the graph of \( f \) at \( x \) and is given by

\[
IROC = m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

provided that the limit exists. Its units of measurement are units of \( f \) per unit of \( x \).

*** \( f'(x) \) is the notation for the derivative of \( f \) and is read “\( f \) prime of \( x \).”****

**Ex**: Given \( f(x) = x^2 + 2 \), answer the following.

(a) Find \( f'(x) \) and use it to determine \( f'(-4), f'(-2), f'(0), f'(1), \) and \( f'(5) \).

(b) Knowing that the derivative of a function gives the slope of the tangent line to the curve, does the result in (a) indicate that \( f(x) \) is increasing or decreasing? Explain.

(c) Find the equation of the tangent line to \( f(x) \) at \( x = 1 \).
Ex: If \( f(x) = 3x^3 + 5x - 1 \) and the IROC of \( f(x) \) is 5 at \( x = 0 \), find the equation of the tangent line to \( f(x) \) at \( x = 0 \).

Ex: The FrezMore Co. has determined that the weekly cost (in dollars) of producing \( x \) dorm room-sized refrigerators is modeled by
\[
C(x) = 0.5x^2 + 3x + 15 \quad 0 \leq x \leq 300
\]

(a) Find \( C(40) \) and interpret.

(b) Find \( C'(40) \) and interpret.