4.2 - Derivatives of Logarithmic Functions

• For \( f(x) = \ln x \), with \( x > 0 \), \( f'(x) = \frac{1}{x} \)

• **Chain Rule for Natural Logarithm:** If \( g \) is a differentiable function of \( x \) and the range of \( g \) is \((0, \infty)\), then the derivative of \( h(x) = \ln[g(x)] \) is

\[
h'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}
\]

• **Chain Rule for General Logarithm:** If \( g \) is a differentiable function of \( x \) and the range of \( g \) is \((0, \infty)\), then the derivative of \( h(x) = \log_b[g(x)] \) is

\[
h'(x) = \left( \frac{1}{\ln b} \right) \left( \frac{1}{g(x)} \right) \cdot g'(x) = \frac{g'(x)}{g(x)(\ln b)}
\]

**Ex:** Differentiate the following:

(a) \( f(x) = 4 \ln x + 3 \)

(b) \( g(x) = \ln x^5 \)

(c) \( h(x) = \left( \frac{2}{x^4} \right) \ln x \)

(d) \( k(x) = \ln(3x^2 + 10) \)

(e) \( y = (\ln x)^5 \)
(f) \( f(x) = \frac{\ln (x^4 - 8x)^2}{x^5 - 6x + 1} \)

(g) \( g(x) = \log_7 x \)

(h) \( h(x) = \ln (\log_5 (2x^3)) \)

(i) \( y = \log_6 \left( \frac{x + 1}{x - 2} \right) \)
Ex: Find the equation of the tangent line to $f(x) = (\ln x^2)(\ln x)^3$ at $x = e$. 