5.1 - First Derivatives and Graphs

**Increasing and Decreasing Functions:** On an open interval \((a, b)\) on which \(f(x)\) is differentiable and continuous

(a) \(f(x)\) is **increasing** on \((a, b)\) if \(f'(x) > 0\) on \((a, b)\).

(b) \(f(x)\) is **decreasing** on \((a, b)\) if \(f'(x) < 0\) on \((a, b)\).

(c) \(f(x)\) is **constant** on \((a, b)\) if \(f'(x) = 0\) on \((a, b)\).

**Ex:** Using the graph of \(f(x)\) below, answer the following questions.

![Graph of f(x)](image)

(a) On what intervals is \(f'(x) > 0\) and what does this say about \(f(x)\)?

(b) On what intervals is \(f'(x) < 0\) and what does this say about \(f(x)\)?

(c) On what intervals is \(f'(x) = 0\) and what does this say about \(f(x)\)?
**Def:** A critical value for \( f(x) \) is an \( x \)-value in the domain of \( f(x) \) for which

1. \( f'(x) = 0 \) or
2. \( f'(x) \) is undefined

**Ex:** Find the critical values for the following functions:

(a) \( f(x) = x^3 + 3x^2 - 9x + 3 \)

(b) \( g(x) = \sqrt{x} \)

**First Derivative Test**
Suppose \( x = c \) is a critical value of \( f(x) \).

1. If \( f''(x) \) changes from \((+)\) to \((-)\) at \( x = c \), then we have that \( f(x) \) is \( \nearrow \searrow \) and at \( x = c \) there is a **relative maximum**.

2. If \( f''(x) \) changes from \((-)\) to \((+)\) at \( x = c \), then we have that \( f(x) \) is \( \searrow \nearrow \) and at \( x = c \) there is a **relative minimum**.

3. If the sign of \( f'(x) \) is the same on both sides of \( x = c \), then at \( x = c \) there is neither a relative maximum nor a relative minimum.

*(NOTE: Relative extrema means all relative maxima and relative minima. All relative extrema will occur at critical values, but not all critical values will produce relative extrema.)*

**Ex:** Determine the intervals where the following functions are increasing and decreasing and locate any **points** where relative extrema occur.

(a) \( f(x) = x^3 + 3x^2 - 9x + 3 \)
(b) \( f(x) = x e^{2x} \)

(c) \( f(x) = x + \ln x \)

(d) \( f(x) = \frac{x^2}{x - 1} \)
Ex: Given $f'(x) = p(x + 2)^2(x - 5)^3(x - 10)$ where $p$ is a function that is always positive, find all critical values of $f(x)$, all intervals where $f(x)$ is increasing and decreasing, and all places where relative extrema occur.