5.4 - Optimizing Functions on a Closed Interval

Def: An **absolute maximum** is the largest value a function obtains on its domain. An **absolute minimum** is the smallest value a function obtains on its domain.

**Extreme Value Theorem**: If a function is *continuous* on the *closed interval* \([a, b]\), then the function must have both an absolute maximum and an absolute minimum on \([a, b]\).
Locating Absolute Extrema of $f(x)$ on $[a, b]$:
1. Verify $f(x)$ is continuous on $[a, b]$.
2. Determine the critical values of $f(x)$ in $(a, b)$.
3. Evaluate $f(x)$ at the critical values and at the endpoints on the interval.
4. The largest value obtained from the previous step is the absolute maximum of $f(x)$ on $[a, b]$ and the smallest value obtained is the absolute minimum of $f(x)$ on $[a, b]$.

Ex: Find the absolute extrema of the following functions on the given intervals.
(a) $f(x) = x^3 - 3x^2 + 2$ on $[-2, 1]$

(b) $f(x) = -8x^3 + 6x - 1$ on $[-1, 1]$

(c) $f(x) = \sqrt[3]{x^2}$ on $[-1, 8]$
Solving Applied Optimization Problems on a Closed Interval:

1. Draw a picture representing the problem, if possible.
2. Select variables to represent the quantity to be maximized or minimized and all other unknowns.
3. Write an equation for the quantity to be optimized. If necessary, eliminate extra variables so that the quantity to be optimized is a function of one variable.
4. Take note of, or determine, the interval of values to be used in order to optimize your function.
5. Apply the process of determining absolute extrema on a closed interval.
6. Answer the question posed in the problem.

Ex: The Camp-n-Swim campgrounds has a small lake for recreational swimming. Periodically, the lake is treated to control the growth of harmful bacteria. The concentration of harmful bacteria per cubic centimeter is given by

\[ HB(x) = 3x^2 - 42x + 150, \quad 0 \leq x \leq 14 \]

where \( x \) is the number of days after treatment.

(a) How many days after a treatment is the concentration minimized?

(b) What is the minimum concentration?
Ex: The Tool Shack manufactures ratchet sets. The monthly fixed costs are $15,000 and variable costs are $180 for each ratchet set produced. The company estimates that 300 sets can be sold each month if the unit price is $280 and that 20 more units can be sold for each decrease of $14 in the price. The company also has a policy that no fewer than 200 sets and no more than 580 sets can be produced each month.

(a) Determine the number of ratchet sets that should be made and sold in order to maximize monthly profit.

(b) At what price should the ratchet sets be sold in order to achieve maximum monthly profit?