4.4 - Absolute Maxima and Minima

Absolute Maxima and Minima

**Def:** An **absolute maximum** is the largest value a function obtains on its domain. An **absolute minimum** is the smallest value a function obtains on its domain.

**Extreme Value Theorem:** If a function is continuous on the closed interval \([a, b]\), then the function must have both an absolute maximum and an absolute minimum on \([a, b]\).
Locating Absolute Extrema of $f(x)$ on $[a, b]$:

1. Verify $f(x)$ is continuous on $[a, b]$.

2. Determine the critical values of $f(x)$ in $(a, b)$.

3. Evaluate $f(x)$ at the critical values on the interval and at the endpoints ($a$ and $b$).

4. The largest value obtained from the previous step is the absolute maximum of $f(x)$ on $[a, b]$ and the smallest value obtained is the absolute minimum of $f(x)$ on $[a, b]$.

**Ex:** Find the absolute extrema of the following functions on the given intervals.

(a) $f(x) = x^3 - 3x^2 + 2$ on $[-2, 1]$

(b) $f(x) = -8x^3 + 6x - 1$ on $[-1, 1]$

(c) $f(x) = \sqrt[3]{x^2}$ on $[-1, 8]$
Second Derivative and Extrema

Find the absolute extrema of the given functions.

Second Derivative Test for Absolute Extrema: For a continuous function, $f$, on any interval, $I$ (open, closed, or half-open), if $x = c$ is the only critical value in the interval where $f'(c) = 0$, and if $f''(c)$ exists, then $f(c)$ is

1. an absolute minimum on $I$ if $f''(c) > 0$
2. an absolute maximum on $I$ if $f''(c) < 0$

If $f''(c) = 0$, then this test fails.
Ex: Determine the absolute extrema of the following functions on the given intervals.

(a) \( f(x) = 4x + \frac{2}{x} \) on \((0,10)\)

(b) \( f(x) = -2x - \frac{9}{x} \) on \((0,20)\)