5.3 - Logarithmic Functions and Their Derivatives

• For \( f(x) = \ln x \), with \( x > 0 \), \( f'(x) = \frac{1}{x} \)

• **Chain Rule for Natural Logarithm:** If \( g \) is a differentiable function of \( x \) and the range of \( g \) is \((0, \infty)\), then the derivative of \( h(x) = \ln [g(x)] \) is

\[
h'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}
\]

• **Chain Rule for General Logarithm:** If \( g \) is a differentiable function of \( x \) and the range of \( g \) is \((0, \infty)\), then the derivative of \( h(x) = \log_b [g(x)] \) is

\[
h'(x) = \left(\frac{1}{\ln b}\right) \cdot \left(\frac{1}{g(x)}\right) \cdot g'(x) = \frac{g'(x)}{g(x)(\ln b)}
\]

**Ex:** Differentiate the following:

(a) \( f(x) = 4 \ln x + 3 \)

(b) \( g(x) = \ln x^5 \)

(c) \( h(x) = \left(\frac{2}{x^4}\right)(\ln x) \)

(d) \( k(x) = \ln(3x^2 + 10) \)

(e) \( y = (\ln x)^5 \)
(f) \( f(x) = \frac{\ln (x^4 - 8x)^2}{x^5 - 6x + 1} \)

(g) \( g(x) = \log_7 x \)

(h) \( h(x) = \ln (\log_5 (2x^3)) \)

(i) \( y = \log_6 \left( \frac{x + 1}{x - 2} \right) \)
**Ex:** Find the equation of the tangent line to \( f(x) = (\ln x^2)(\ln x)^3 \) at \( x = e \).

**Ex:** On a national tour of a small new rock band, the demand for T-shirts is given by

\[
p = 15 - 4\ln x \quad 1 \leq x \leq 40
\]

where \( x \) is the number of T-shirts that can be sold during a single concert at a price of \( \$p \). If the shirts cost the band \$5 each, how should they be priced in order to maximize the profit per concert?
Ex: Using the graphing strategies learned in this class, summarize the pertinent information and graph \( f(x) = (\ln x)^2 \).