The exam consists of 19 questions, the first 15 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 15 mark your answers on the ScanTron form.

1. [4 pts] Given $\mathbf{a} = \langle -1, 2 \rangle$ and $\mathbf{b} = \langle 5, -3 \rangle$, find $3\mathbf{a} + \mathbf{b}$.

   (a) $\langle -8, 9 \rangle$
   (b) $\langle 4, -1 \rangle$
   (c) $\langle -3, 6 \rangle$
   (d) $\langle 2, 3 \rangle$
   (e) $\langle 5, 0 \rangle$

2. [4 pts] Given the function $f(x) = (\ln(x^4 + 1))^2$, find $f'(1)$.

   (a) 4
   (b) 0
   (c) 1
   (d) $\ln 4$
   (e) $\ln 16$
3. [4 pts] Find $\int e^{2x}(1 + e^{2x})^3 \, dx$.

(a) $e^{8x} + C$

(b) $e^{2x}(1 + e^{2x})^4 + C$

(c) $\frac{1}{8}e^{2x}(1 + e^{2x})^4 + C$

(d) $\frac{1}{8}(1 + e^{2x})^4 + C$

(e) $\frac{1}{4}(1 + e^{2x})^4 + C$

4. [4 pts] Find an equation for the line tangent to the curve $\mathbf{r}(t) = \langle \sin t, t^2 + t + 1 \rangle$ at the point corresponding to $\mathbf{r}(0)$.

(a) $y = x + 1$

(b) $y = 1$

(c) $x = 0$

(d) $y = 2x + 3$

(e) $y = 3x - 1$
5. [4 pts] Use Newton’s method to find a second approximation \( x_2 \) to a root of the equation 
\[ 5x^2 - e^x = 0 \]
given the initial approximation \( x_1 = 1 \).

(a) \( \frac{5}{10 - e} \)
(b) 1
(c) \( 5 - 2e \)
(d) \( \frac{5}{9} \)
(e) \( \frac{5 - e}{10 - e} \)

6. [4 pts] Find the value of \( c \) which makes the function

\[
f(x) = \begin{cases} 
\sin(cx) & \text{if } x < 0 \\
\frac{e^{-x} - 1}{x} & \text{if } x \geq 0 \\
2 + \int_0^x \tan^{-1}(e^t) \, dt & \text{if } x \geq 0 
\end{cases}
\]
continuous everywhere.

(a) \(-2\)
(b) \(-1\)
(c) 0
(d) 1
(e) 2
7. [4 pts] Which of the following is true?

(a) \( \int_0^1 \sqrt{1 + x^4} \, dx < 1 \)
(b) \( \int_0^1 -e^{-x^3} \, dx \geq 0 \)
(c) \( \int_1^2 \tan^{-1}(x^2) \, dx \geq \pi \)
(d) \( \int_{-1}^1 \frac{\sin x}{x^8 + 3} \, dx = 0 \)
(e) \( \int_0^1 x^4 \, dx = \frac{1}{4} \)

8. [4 pts] Find the derivative of the function \( f(x) = \tan^{-1}(e^{x^2}) \).

(a) \( \frac{2xe^{x^2}}{1 + e^{2x^2}} \)
(b) \( \frac{2xe^{x^2}}{1 + e^{2x^2}} \)
(c) \( \frac{4x}{1 + e^{2x^2}} \)
(d) \( \frac{2x}{1 + e^{2x^2}} \)
(e) \( \frac{2xe^{2x^2}}{1 + e^{2x^2}} \)
9. [4 pts] A horizontal force of 20 N is applied to move a box up a ramp that is 10 m long and inclined at an angle of 30 degrees to the horizontal. What is the work done on the box?

(a) 200 J
(b) 100√3 J
(c) 0 J
(d) 100 J
(e) 20 J

10. [4 pts] In which of the following intervals does the equation \( x^4 + 8x - 1 = 0 \) have a solution?

(a) \((-2, -1)\)
(b) \((-1, 0)\)
(c) \((0, 1)\)
(d) \((1, 2)\)
(e) \((2, 3)\)

11. [4 pts] Let \( y \) be defined implicitly in terms of \( x \) by the equation \( x^2e^y - y^2 = 1 \). Find \( y' \) when \((x, y) = (1, 0)\).

(a) 0
(b) \(-\frac{1}{2}\)
(c) -1
(d) -2
(e) \(e\)
12. [4 pts] Find the derivative of the function \( f(x) = \frac{(x + 4)^{24}(x - 3)^8}{(x + 3)^{57}(x + 6)^5} \).

(a) \( \frac{24(x + 4)^{23}(x - 3)^8 + 8(x + 4)^{24}(x - 3)^7}{(x + 3)^{56}(x + 6)^4} \)

(b) \( \frac{24(x + 4)^{23}(x - 3)^8 + 8(x + 4)^{24}(x - 3)^7}{(x + 3)^{56}(x + 6)^4} f(x) \)

(c) \( \left( \frac{24}{x + 4} + \frac{8}{x - 3} - \frac{57}{x + 3} - \frac{5}{x + 6} \right) f(x) \)

(d) \( \left( \frac{1}{24(x + 4)} + \frac{1}{8(x - 3)} - \frac{1}{57(x + 3)} - \frac{1}{5(x + 6)} \right) f(x) \)

(e) \( \frac{24(x + 4)^{23}(x - 3)^8 + 8(x + 4)^{24}(x - 3)^7}{(x + 3)^{114}(x + 6)^{10}} \)

13. [4 pts] Find the linear approximation to the function \( f(x) = (x + 2)e^{x-1} \) at the point \( x = 1 \).

(a) \( L(x) = 2ex - 2e + 3 \)

(b) \( L(x) = 3x - 2 \)

(c) \( L(x) = 3x + 3 \)

(d) \( L(x) = 4ex - e \)

(e) \( L(x) = 4x - 1 \)
14. [4 pts] Suppose that \( f'(x) = x^2 - 4x + 15 \) on the interval \((1, 3)\). What can be said about \( f \)?

(a) \( f \) has a local minimum at some point in \((1, 3)\)

(b) \( f \) has a local maximum at some point in \((1, 3)\)

(c) the graph of \( f \) is concave upward on \((1, 3)\)

(d) the graph of \( f \) is concave downward on \((1, 3)\)

(e) the graph of \( f \) has a point of inflection \((c, f(c))\) for some \( c \) in \((1, 3)\)

15. [4 pts] Given the function \( f(x) = \int_x^{2x} \frac{1}{t^2 + t^2 + 1} \, dt \), find \( f'(0) \).

(a) \( \frac{1}{3} \)

(b) \(-\frac{1}{3}\)

(c) 0

(d) 1

(e) 2
16. [10 pts] Compute \( \int_{0}^{3} \left( 2x + \frac{3x}{\sqrt{x + 1}} \right) \, dx. \)
17. [10 pts] Find an equation for the tangent line to the graph of the function \( f(x) = 2 \sin^2 x \) at \( x = \frac{\pi}{4} \).
18. [10 pts] Two cars start moving away from an intersection. Car A travels due north and its distance from the intersection after $t$ seconds is $(t + 1)^2 - 1$ meters. Car B travels due south and its speed after $t$ seconds is $\frac{2t}{t^2 + 1}$ meters per second. How fast is the distance between the two cars increasing when $t = 1$?
19. [10 pts] Find the largest possible area of a rectangle whose base lies on the $x$-axis and whose two vertices above the $x$-axis lie on the curve $y = 80 - x^4$. 