The exam consists of 19 questions, the first 15 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 15 mark your answers on the ScanTron form.

1. [4 pts] Given $a = \langle 2, -1 \rangle$ and $b = \langle -3, 2 \rangle$, find $a + 2b$.

   (a) $\langle 8, -3 \rangle$
   (b) $\langle -4, 1 \rangle$
   (c) $\langle 1, 0 \rangle$
   (d) $\langle -1, 1 \rangle$
   (e) $\langle -4, 3 \rangle$

2. [4 pts] Which of the following vectors is orthogonal to $\langle 4, -5 \rangle$?

   (a) $\langle 3, -2 \rangle$
   (b) $\langle 3, 2 \rangle$
   (c) $\langle 5, 4 \rangle$
   (d) $\langle 4, -5 \rangle$
   (e) $\langle 5, -4 \rangle$
3. [4 pts] Find an equation for the line tangent to the curve $r(t) = \langle t^2, 2t^3 + 1 \rangle$ at the point corresponding to $r(1)$.

(a) $y = 2x + 1$
(b) $y = 3x$
(c) $y = 3$
(d) $y = 2x + 3$
(e) $y = 3x - 3$

4. [4 pts] Which of the following is a horizontal asymptote for the function $f(x) = \frac{x^2 + 1}{3 - x^2}$?

(a) $y = 0$
(b) $y = 1$
(c) $y = -1$
(d) $y = \frac{1}{3}$
(e) $y = 3$

5. [4 pts] Suppose that $f$ and $g$ are integrable functions such that $\int_0^2 f(x) \, dx = 3$, $\int_1^2 g(x) \, dx = 4$, and $\int_0^2 [f(x) + g(x)] \, dx = 2$. Find $\int_0^1 g(x) \, dx$.

(a) $-5$
(b) $-1$
(c) 0
(d) 2
(e) 7
6. [4 pts] Find the value of $c$ which makes the function

$$f(x) = \begin{cases} 
\frac{e^{cx} - 1}{x + \sin x} & \text{if } x > 0 \\
\frac{1}{\cos x} & \text{if } x \leq 0 
\end{cases}$$

continuous everywhere.

(a) $-2$
(b) $-1$
(c) $0$
(d) $1$
(e) $2$

7. [4 pts] Find the derivative of the function $f(x) = \frac{e^x(x + 2)^9}{(x - 1)^5(x + 3)^{11}}$.

(a) \[
\frac{e^x(x + 9)(x + 11)}{(x - 1)^{10}(x + 3)^{22}}
\]

(b) \[
\left(1 + \frac{9}{x + 2} - \frac{5}{x - 1} - \frac{11}{x + 3}\right) f(x)
\]

(c) \[
\left(1 + \frac{1}{(x + 2)^9} - \frac{1}{(x - 1)^5} - \frac{1}{(x + 3)^{11}}\right) f(x)
\]

(d) \[
\left(e^x + \frac{1}{9(x + 2)^8} - \frac{1}{5(x - 1)^4} - \frac{1}{11(x + 3)^{10}}\right) f(x)
\]

(e) \[
\left(e^x + \frac{1}{9(x + 2)} - \frac{1}{5(x - 1)} - \frac{1}{11(x + 3)}\right) f(x)
\]
8. [4 pts] In which of the following intervals does the equation $x^4 = 3 - x$ have a solution?

(a) $(-1, 0)$
(b) $(0, 1)$
(c) $(1, 2)$
(d) $(2, 3)$
(e) $(3, 4)$

9. [4 pts] Given the function $f(x) = \int_x^{x^2} \frac{e^t}{1 + t^2} dt$, find $f'(0)$.

(a) $-1$
(b) $0$
(c) $e$
(d) $\frac{1}{2}$
(e) $-\frac{1}{2}$

10. [4 pts] Compute $\lim_{x \to 0} (1 + x^2)^{1/x^2}$.

(a) $e$
(b) $\frac{1}{2}$
(c) $0$
(d) $1$
(e) $2$
11. [4 pts] Compute $\int_{-1}^{2} (4x^3 + 3) \, dx$.

(a) 20  
(b) 22  
(c) 24  
(d) 26  
(e) 28

12. [4 pts] Let $y$ be defined implicitly in terms of $x$ by the equation $\tan^{-1}(xy + 1) = x$. Find $y'$ when $(x, y) = (\frac{\pi}{4}, 0)$.

(a) $\frac{8}{\pi}$  
(b) $\frac{4}{\pi}$  
(c) 2  
(d) $-2$  
(e) $e$

13. [4 pts] Given the function $f(x) = x^2 + e^x + \tan^{-1}(x)$, find $(f^{-1})'(1)$.

(a) 0  
(b) $\frac{1}{2}$  
(c) 1  
(d) 2  
(e) $e + \frac{5}{2}$
14. [4 pts] Suppose that $f$ is a function such that $f'(x) = 2x^3 - 3x^2 - 1$ on the interval $(0, 2)$. What can be said about $f$?

(a) $f$ has a local minimum at some point in $(0, 2)$
(b) $f$ has a local maximum at some point in $(0, 2)$
(c) the graph of $f$ is concave upward on $(0, 2)$
(d) the graph of $f$ is concave downward on $(0, 2)$
(e) $f$ is increasing on $(0, 2)$

15. [4 pts] Find the derivative of the function $f(x) = e^{\tan^{-1}(x^2)}$.

(a) $\frac{e^{\tan^{-1}(x^2)}}{1 + x^2}$
(b) $\frac{e^{\tan^{-1}(x^2)}}{1 + x^4}$
(c) $\frac{2x}{1 + x^4}$
(d) $\frac{2xe^{\tan^{-1}(x^2)}}{1 + x^4}$
(e) $2xe^{\tan^{-1}(x^2)}$
16. [10 pts] Find the absolute maximum value of the function \( f(x) = x^3 - 3x + 2 \) on the interval \([0, 2]\).
17. [10 pts] Find an equation for the tangent line to the graph of the function

\[ f(x) = \sin x + \int_{\pi}^{x} \frac{1}{1 + t^4} \, dt \]

at \( x = \pi \).
18. [10 pts] A farmer wishes to fence off a rectangular area in a field and also to subdivide this area into four equal rectangular pieces as pictured. If the large rectangle is required to be 160 m$^2$, what is the minimum length of fence needed to do this?
19. [10 pts] A ladder of length 10 ft leans against a vertical wall. The bottom of the ladder slips in such a way that after $t$ seconds its distance from the wall is $(t^2 + 2)$ ft. How fast is the top of the ladder sliding down the wall when $t = 2$?