Section 3.1 - Graphing Systems of Linear Inequalities

Example 1: Find the graphical solution of the inequality \( y - x \leq 0 \).

Example 2: Find the graphical solution of the inequality \( 5x - 3y < 15 \).
How to find the solution set of a system of linear inequalities:

1. Solve each inequality for y. (Remember: If you divide by a negative number, the inequality sign reverses)
2. Replace the inequality with an equal sign and graph each corresponding line.
3. ”Cross out” or shade the false region. This can be done on the calculator by arrowing to the left of Y1 and hitting enter until you get the appropriate triangle.
4. The unshaded region is the solution set (feasible region).

Example 3: Determine graphically the solution set for the system of inequalities:

\[
2x + 4y > 16
\]

\[
-x + 3y \geq 7
\]
Example 4: Determine graphically the solution set for the system of inequalities:

\[ x + y \leq 4 \]
\[ 2x + y \leq 6 \]
\[ 2x - y \geq -1 \]
\[ x \geq 0, y \geq 0 \]

Definition: A solution set of a system of linear inequalities is **bounded** if it can be enclosed by a circle. Otherwise, it is **unbounded**.
Section 3.2 - Linear Programming Problems

Definition: A linear programming problem consists of a linear objective function to be maximized or minimized subject to certain constraints in the form of linear equalities or inequalities.

Example 1: A farmer plans to plant two crops, A and B. The cost of cultivating crop A is $50/acre, whereas that of crop B is $20/acre. The farmer has a maximum of $4,000 available for land cultivation. Each acre of crop A requires 75 labor-hours, and each acre of crop B requires 150 labor-hours. The farmer has a maximum of 15,000 labor-hours available. If she expects to make a profit of $90/acre on crop A and $100/acre on crop B, how many acres of each crop should she plant in order to maximize her profit? (Set-up the Linear Programming Problem)

Example 2: Deluxe River Cruises operates a fleet of river vessels. The fleet has two types of vessels: A type-A vessel has 60 deluxe cabins and 96 standard cabins, whereas a type-B vessel has 40 deluxe cabins and 160 standard cabins. Under a charter agreement with Odyssey Travel Agency, Deluxe River Cruises is to provide Odyssey with a minimum of 240 deluxe and 672 standard cabins for their 15-day cruise in May. It costs $44,000 to operate a type-A vessel and $54,000 to operate a type-B vessel for that period. How many of each type vessel should be used in order the keep the operating costs to a minimum? (Set-up the Linear Programming Problem)
Example 3: Steinwelt Piano manufactures pianos in two plants: Plant I and Plant II. The output of Plant I is at most 300 per month, whereas the maximum output of Plant II is 250 per month. The pianos are shipped to two warehouses that serve as distribution centers for the company. To fill current and projected orders, Warehouse A requires a minimum of 200 pianos per month and the minimum monthly requirement at Warehouse B is 150 pianos. The shipping cost, for each piano, from the plants to the warehouses is shown in the table.

<table>
<thead>
<tr>
<th></th>
<th>Plant I</th>
<th>Plant II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse A</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Warehouse B</td>
<td>70</td>
<td>50</td>
</tr>
</tbody>
</table>

What shipping schedule will enable the company to meet the warehouses’ requirements while keeping shipping costs to a minimum?
Section 3.3 - Method of Corners

Example 1: Let’s look at solving Example 1 from Section 3.2. We found that the linear programming problem was:

Maximize \( P = 90x + 100y \)
Subject to
\[
egin{align*}
50x + 20y & \leq 4,000 \\
75x + 150y & \leq 15,000 \\
x & \geq 0, y \geq 0
\end{align*}
\]
Method of Corners:

1. Graph the corresponding equations by hand or on your calculator (i.e. solve each inequality for \( y \) and replace the inequality sign with an equal sign). Be careful in choosing your window. If there are non-negativity constraints on \( x \) and \( y \), account for these by choosing \( x\)-min and \( y\)-min to be 0.

2. Determine the feasible region. This is done by ”crossing out” or shading the false region for each inequality. This is done on the calculator by arrowing to the left of Y1 and hitting enter until you get the appropriate triangle. The unshaded region is the feasible region.

3. Find the coordinates of the corner points. You can either do this algebraically or on the calculator. There are four possible corner points:
   - The origin (i.e the point \((0,0)\))
   - The intersection of two lines (use 2nd TRACE 5:intersect)
   - The intersection of a line and the \( y\)-axis (recognize that this is the \( y\)-intercept of the line or use 2nd TRACE 1:value and enter in 0 for \( x \))
   - The intersection of a line and the \( x\)-axis (plug in 0 for \( y \) and solve for \( x \) or use 2nd TRACE 2:zero and specify a left and right bound that includes the point where the line actually crosses the \( x\)-axis)

4. Make a table and evaluate the objective function at each corner point.

Example 2: Let’s solve Example 2 from Section 3.2. We found that the linear programming problem was:

Minimize \[ C = 44,000x + 54,000y \]
Subject to \[ 60x + 40y \geq 240 \]
\[ 96x + 160y \geq 672 \]
\[ x \geq 0, y \geq 0 \]
Example 3: You manage an ice cream factory that makes two flavors: Creamy Vanilla and Continental Mocha. Each quart of Creamy Vanilla requires 2 eggs and 3 cups of cream. Each quart of Continental Mocha requires 1 egg and 3 cups of cream. You have in stock 500 eggs and 900 cups of cream. You make a profit of $3 on each quart of Creamy Vanilla and $4 on each quart of Continental Mocha.

a) How many quarts of each flavor should you make in order to earn the largest profit?

b) Are there any leftover resources? Be specific.
Example 4: Solve the following linear programming problem

Minimize \[ P = 2x + 2y \]
Subject to \[ 2x + 3y \leq 30 \]
\[ y - x \leq 5 \]
\[ x + y \geq 5 \]
\[ x \leq 10 \]
\[ x \geq 0, y \geq 0 \]
Example 5: