Fall 2011
Week-in-Review #2

Section 2.1

- When solving a system of linear equations, the solution will either be a unique solution, no solution, or infinitely many solutions.
- When setting up a word problem, always make sure to define your variables first!

1. Solve the following systems of linear equations using any method.
   a) \(2x - y = 1\)  
   b) \(-2x + 3y = 21\)  
   c) \(x - 5y = 15\)
   
   \[5x + y = 27\]  
   \[4x - 6y = 12\]  
   \[-3x + 15y = -45\]

2. Determine the value of \(k\) for which the system of linear equations has no solution.
   \[2x + 5y = 7\]
   \[3x + ky = 15\]

3. Jack and Jill decide to go into the business of making table decorations for weddings. They will be decorating three different sizes of tables. A small table seats 4 people, a medium table seats 8 people, and a large table seats 12 people. For a certain wedding of 344 people, the bride has decided her table decorating budget is $2,520 and she wants twice as many medium tables as small and large tables combined. If it cost $40 to decorate a small table, $60 to decorate a medium table, $75 to decorate a large table, how many tables of each size should they use? (Set-up but do not solve.)

4. A theater has a seating capacity of 1200 and charges $3 for children, $4 for students, and $5 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled $5100. How many children, students, and adults attended the show? (Set-up but do not solve.)

5. I want to make three different types of cakes. The recipe to make 1 pineapple cake calls for 1 cup of flour, 2 cups of sugar, and 3 cups of water. The recipe to make 1 chocolate cake calls for 2 cups of flour, 1 cup of sugar, and 3 cups of water. The recipe to make 1 lemon cake calls for 1 cup of flour, 2 cups of sugar, and 3 cups of water. The recipe to make 1 lemon cake calls for 1 cup of flour, 2 cups of sugar, and 3 cups of water. Assume that the entire budget is used and there will be no extra seats.

6. Is the following augmented matrix in row-reduced form? If not, explain why not.
   \[
   \begin{bmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 1 & 0 \\
   \end{bmatrix}
   \]

   \[
   \begin{bmatrix}
   1 & 3 & 0 & 0 & 0 \\
   0 & 0 & 1 & 0 & 0 \\
   0 & 0 & 0 & 0 & 1 \\
   \end{bmatrix}
   \]

   \[
   \begin{bmatrix}
   1 & 3 & 0 & 0 & 0 \\
   0 & 0 & 1 & 3 & 0 \\
   0 & 0 & 4 & 2 & 0 \\
   \end{bmatrix}
   \]

7. Solve the system of linear equations using Gauss-Jordan Elimination by hand.
   \[9y = -3x - 15\]
   \[2x = -8y + 6\]

8. Solve the system we set-up in problem 3 using Gauss-Jordan Elimination by hand.

9. Pivot the given system about the boxed element.
   \[
   \begin{bmatrix}
   1 & -3 & -4 & 17 \\
   0 & 7 & 2 & -8 \\
   0 & 4 & 5 & 9 \\
   \end{bmatrix}
   \]

10. Find the solution(s) to the following systems of linear equations using any method.
    \[
    \begin{align*}
    (a) & \quad 2x + 2y - z = 7 \\
    & \quad 2x - y - 3z = 3 \\
    & \quad 3y + 2z = 4 \\
    \end{align*}
    \]
    \[
    \begin{align*}
    (b) & \quad 6z - 3x = 3y + 9 \\
    & \quad 2x - y + 3z = 7 \\
    & \quad x = 2y - 5z \\
    \end{align*}
    \]
    \[
    \begin{align*}
    (c) & \quad 2x_1 - x_2 + 5x_3 + 2x_4 = 2 \\
    & \quad x_1 - 2x_2 + 5x_3 - 3x_4 = 4 \\
    \end{align*}
    \]

Sections 2.2, 2.3

- The goal of the Gauss-Jordan Elimination Method is to get the augmented matrix into Row Reduced Form. A matrix is in Row Reduced Form when:
  
  (a) Each row of the coefficient matrix consisting entirely of zeros lies below any other row having nonzero entries.
  
  (b) The first nonzero entry in each row is 1 (called a leading 1)
  
  (c) In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
  
  (d) If a column contains a leading 1, then the other entries in that column are zeros.

  **Note:** We only consider the coefficient side (left side) of the augmented matrix when determining whether the matrix is in row-reduced form.

- To put a matrix into Row Reduced Form, there are three valid Row Operations:

(a) Interchange any two rows \((R_i \leftrightarrow R_j)\)

(b) Replace any row by a nonzero constant multiple of itself \((cR_i)\)

(c) Replace any row by the sum of that row and a constant multiple of any other row \((R_i + cR_j)\).

- If the problem does not specify that you must do it by hand, you can use the \texttt{rref} command on your calculator:
  
  - First, enter the matrix into the calculator. \(2nd x^{-1}\) and then arrow to \texttt{EDIT}
  
  - Go back to the home screen and then back to the matrix menu \(2nd x^{-1}\) and then arrow to \texttt{MATH}
  
  - Select option \texttt{B:rref}
  
  - Call the matrix by going back to the matrix menu \(2nd x^{-1}\)