Math142 Lecture Notes
1.2 - Linear Functions and Average Rate of Change

Definition
A linear function has the form \( f(x) = mx + b \) where \( m \) is the average rate of change, or slope of the line, and \( b \) is a constant (\( m \) and \( b \) are real numbers).

The slope (average rate of change) of a line, denoted by \( m \), is a measurement of the steepness of the line. Given two points on a line \( (x_1, y_1) \) and \( (x_2, y_2) \), the slope of the line is computed by
\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The slope of the line also gives the average rate of change of \( y \) with respect to \( x \).

Example 1:
(a) Find the average rate of change between \( (3, -2) \) and \( (5, -6) \).

(b) If the independent variable value increases by 2 units, how will this affect the dependent value?

Definition
The cost \( C(x) \) of producing \( x \) units of a product is given by the cost function
\[
C(x) = \left( \text{variable costs} \right) \cdot \left( \frac{\text{units produced}}{\text{costs}} \right) + \left( \text{fixed costs} \right)
\]

Example 2: The PPPP (Pittsburgh Plasma Producing Plant) has fixed costs of $225,000 and the materials and labor to manufacture each set run $3500. (Note: The Pittsburgh Plant only makes the 32” set.) Let \( x \) represent the number of 32” sets made and sold, and write a cost function for the \( P^3 \) Plant.

Example 3: The Huntsville Plasma TV Producing Plant has total costs of $975,000 to produce 250 plasma television sets. If they have fixed costs of $175,000, find the cost function for Huntsville Plant.
**Equations of Lines**

<table>
<thead>
<tr>
<th>Type of Line</th>
<th>Value of m</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing</td>
<td>Positive</td>
<td><img src="image" alt="Increasing Line Diagram" /></td>
</tr>
<tr>
<td>Decreasing</td>
<td>Negative</td>
<td><img src="image" alt="Decreasing Line Diagram" /></td>
</tr>
<tr>
<td>Horizontal</td>
<td>Zero</td>
<td><img src="image" alt="Horizontal Line Diagram" /></td>
</tr>
<tr>
<td>Vertical</td>
<td>Undefined</td>
<td><img src="image" alt="Vertical Line Diagram" /></td>
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</tbody>
</table>

- **slope-intercept form**  
  \[ y = mx + b \] where \( m \) is the slope and \( b \) is the \( y \)-intercept.

- **point-slope form**  
  \[ y - y_1 = m(x - x_1) \] where \( m \) is the slope and the line passes through \((x_1, y_1)\).

- **calculator friendly form (CF)**  
  \[ y = m(x - x_1) + y_1 \] where \( m \) is the slope and the line passes through \((x_1, y_1)\).

- **vertical line**  
  \[ x = c \] where \( c \) is a constant.

- **horizontal line**  
  \[ y = c \] where \( c \) is a constant.

**Example 4:** Write an equation of line and determine whether it is increasing or decreasing.

(a) The line contains the points \((2, 6)\) and \((5, -3)\) (point-slope form).

(b) The line has an \( x \)-intercept at 2 and a \( y \)-intercept at 4 (CF form).

(c) The horizontal line that passes through \((-2, 3)\).

(d) The line that passes through \((-1, 4)\) with a slope of \( \frac{2}{3} \) (slope intercept).

(e) A linear function \( f \) in which \( f(5) = 1 \) and \( f(-7) = -1 \) (CF form).
Example 5: Determine the $x-$ and $y-$ intercepts of $f(x) = \frac{2}{3}(x + 1) + 3$.

Example 6: A XQ-383 sports car costs $40,000 and depreciates $3000 per year.

(a) Determine an equation for the depreciation function.

(b) How much will the car be worth in 5 years?

Definition
The cost of producing the $(x + 1)^{st}$ item of a product is called the **marginal cost**.

Example 7: The ProAudio Company manufactures DVD disks. It determines that the weekly fixed costs are $14,000 and the variable costs are $2.60 per disk.

(a) Determine the linear cost function $C$ and interpret $C(1500)$.

(b) Identify the marginal cost. At a 1500 per week production level, what is the cost of manufacturing the 1501$^{st}$ disk?

Example 8: A taxi charges a flat fee of $3 up to the first mile driven and an additional $0.25 for 1 mile and each addition mile afterward. Write and graph a function that represents this.
Example 9: For the piecewise-defined function $f(x) = \begin{cases} 
  x + 2 & x < 1 \\
  3 - x & x \geq 1
\end{cases}$

(a) Evaluate $f(0), f(1), \text{ and } f(3)$.

(b) Make an accurate graph of the function.

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**Definition**

The **absolute value function** $f(x) = |x|$ is defined by

$$f(x) = \begin{cases} 
  -x & x < 0 \\
  x & x \geq 0
\end{cases}$$

Example 10: Graph $f(x) = |x|$

Example 11: Rewrite the $f(x) = |2 - x|$ in piecewise form and then graph it.