Math142 Lecture Notes
1.4 - Operations on Functions

Operations on Functions
Let $f$ and $g$ be functions. Then, for all $x$-values for which both $f$ and $g$ exist, we have

1) $(f + g)(x) = f(x) + g(x)$  
2) $(f - g)(x) = f(x) - g(x)$

3) $(f \cdot g)(x) = f(x) \cdot g(x)$  
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Example 1: Let $f(x) = \sqrt{4 - 2x}$, $g(x) = x^2 - 9$ and $h(x) = 5 - 5x$, find

(a) $(h + g)(x)$ and its domain.

(b) $(g - h)(x)$ and its domain.

(c) $(g \cdot h)(x)$ and its domain.

(d) $\left(\frac{f}{g}\right)(x)$ and its domain.

Example 2: The Maximum Monitor sells 19” plasma TV’s for $1800. The company has fixed costs of $225,750 and each set costs $925 to manufacture.

(a) Find a cost function for Maximum Monitor.

(b) Find a revenue function for $M^2$.

(c) Find a profit function for $M^2$.

(d) How many sets do they have to sell to break-even?
Previously Discussed Business Functions

- price-demand function: gives the price, \( p(x) \), at which consumers buy exactly \( x \) units of a product.

- Cost Function: \( C(x) = \left( \frac{\text{variable costs}}{\text{production}} \right) \cdot \left( \frac{\text{units produced}}{\text{costs}} \right) + \left( \frac{\text{fixed costs}}{\text{costs}} \right) = mx + b \)

Other Business Functions

- Revenue Function: \( R(x) = \left( \frac{\text{quantity sold}}{\text{price}} \right) \cdot \left( \frac{\text{unit price}}{\text{price}} \right) = x \cdot p(x) \)

- Profit Function: \( P(x) = R(x) - C(x) \quad \text{Profit} = \text{Revenue} - \text{Cost} \)

Example 3: It is known that the demand for a company’s product is 12 units when each product sells for $48 and the demand is 10 units when each product sells for $60. In addition, the company’s fixed costs for production are $240 and the total costs for the company are $300 to produce 3 units. Find:

(a) the company’s price-demand function.

(b) the company’s cost function.

(c) the company’s revenue function.

(d) the company’s profit function.

(e) Find the vertex of \( P(x) \) and interpret each coordinate.
The functions found in Example 3 are graphed below.

- Break-even occurs when $R(x) = C(x)$ or when $P(x) = 0$.
- Profit gain occurs when $R(x) > C(x)$ or when $P(x) > 0$.
- Profit loss occurs when $R(x) < C(x)$ or when $P(x) < 0$.

Example 4: Find the break-even points for example 3.

Example 5: Use the graph to the right and determine the interval(s) of $x$ over which the company will realize:

(a) a profit

(b) a loss
Definition

A polynomial function of degree $n$ has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where $a_1, a_2, \ldots, a_n$ are real number constants and $n$ is a whole number.

- The degree of the polynomial function is $n$, the highest power present.
- The domain of any polynomial function is $(-\infty, \infty)$.
- Polynomial functions are always continuous (no breaks or holes).

The End Behavior of Polynomial Functions

<table>
<thead>
<tr>
<th>Even Degree Polynomials</th>
<th>negative leading coeff</th>
<th>positive leading coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td>right side behavior</td>
<td>Falls to $-\infty$</td>
<td>Rises to $\infty$</td>
</tr>
<tr>
<td>$(x \to \infty)$</td>
<td>$(f(x) \to -\infty)$</td>
<td>$(f(x) \to \infty)$</td>
</tr>
<tr>
<td>left side behavior</td>
<td>Falls to $-\infty$</td>
<td>Rises to $\infty$</td>
</tr>
<tr>
<td>$(x \to -\infty)$</td>
<td>$(f(x) \to -\infty)$</td>
<td>$(f(x) \to \infty)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Odd Degree Polynomials</th>
<th>negative leading coeff</th>
<th>positive leading coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td>right side of graph</td>
<td>Falls to $-\infty$</td>
<td>Rises to $\infty$</td>
</tr>
<tr>
<td>$(x \to \infty)$</td>
<td>$(f(x) \to -\infty)$</td>
<td>$(f(x) \to \infty)$</td>
</tr>
<tr>
<td>left side behavior</td>
<td>Rises to $\infty$</td>
<td>Falls to $-\infty$</td>
</tr>
<tr>
<td>$(x \to -\infty)$</td>
<td>$(f(x) \to \infty)$</td>
<td>$(f(x) \to -\infty)$</td>
</tr>
</tbody>
</table>

Leading Coefficient

Positive  | Negative
---|---
Even  |  
Odd  |  

Example 6: Determine the end behavior of

(a) $f(x) = -2x^3 - 7x^2 + 4x - 2$

(b) $g(x) = 2x^4 + 4x^3 - 2x^2 + 1$

c) $h(x) = 5x^3 + 1$

(d) $p(x) = -.2x^8 - 2x^7 + 3x^3 - 1$