Math142 Lecture Notes

1.5 - Rational, Radical, and Power Functions

- Radical/Rational Exponent Functions: $f(x) = \sqrt[b]{[g(x)]^a} = [g(x)]^{a/b}$
  
  Domain: $(-\infty, \infty)$ if $b$ is odd.
  Domain: $x$ values for which $g(x) \geq 0$ if $b$ is even.

Example 1:

(a) Rewrite $f(x) = \sqrt[8]{8 - 2x}$ as a function with rational exponents and then determine its domain.

(b) Rewrite $g(x) = (4x - 9)^{3/5}$ as a radical function and then determine its domain.

Rational Function: $y = \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomials with $g(x) \neq 0$.

- Finding Vertical Asymptotes:
  
  Consider the rational function $h(x) = \frac{f(x)}{g(x)}$ where $f$ and $g$ are polynomials.

  If there is a value $c$ that makes the denominator zero, and NOT the numerator, then the vertical line $x = c$ is a vertical asymptote.

Example 2

(a) Rewrite $f(x) = \frac{x^2 - 3x - 10}{x^2 + 5x + 6}$ in factored form and find any vertical asymptotes.
(b) Rewrite \( f(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3} \) in factored form and find any vertical asymptotes.

(c) Find the vertical asymptotes of \( f(x) = \frac{2x}{x - 3} \)

(d) Find any vertical asymptotes of \( g(x) = \frac{2x^2 - 5x - 3}{x^2 - 16} \)

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**Finding Horizontal Asymptotes:**

Consider the rational function \( h(x) = \frac{f(x)}{g(x)} \) where \( f \) and \( g \) are polynomials.

- If the degree of \( f(x) \) is greater than the degree of \( g(x) \), there is **no** horizontal asymptote.
- If the degree of \( f(x) \) is less than the degree of \( g(x) \), the horizontal asymptote is \( y = 0 \).
- If the degrees are equal, the horizontal asymptote is \( y = \frac{a}{b} \) where \( a \) and \( b \) are the coefficients of the leading coefficients.

Example 3: Find the horizontal asymptotes of

(a) \( g(x) = \frac{2x^2 - 5x - 3}{x^2 - 16} \)

(b) \( h(x) = \frac{3x^3 - 2x + 1}{x^2 + 1} \)

(c) \( p(x) = \frac{2x^3 + 1}{x^4 - 2x^3 + 4x^1 - 1} \)
Example 4: Determine the $x$- and $y$-intercepts of $g(x) = \frac{2x^2 - 5x - 3}{x^2 - 16}$, the horizontal and vertical asymptotes, and sketch a graph of the function.

Example 5: Use a model of the Laffer curve based on the rational function

$$f(x) = \frac{80x - 8000}{x - 110}, \quad 30 \leq x \leq 100$$

where $x$ represents the tax rate percentage and $f(x)$ represents the government tax revenue in tens of billions of dollars.

(a) Evaluate $f(45)$ and interpret

(b) Find the $x$-intercept and interpret.

(c) Find the average rate of change for $x = 45$ and $\Delta x = 30$ and interpret.

- Power Function

A function of the form

$$f(x) = a \cdot x^b$$

is called a power function, where $a$ and $b$ are real numbers.
Positive Exponent Power Functions

\[ f(x) = a \cdot x^b, \quad \text{with } a > 0 \]

Example 6: The growth of males in the country of Brazil can be modeled by

\[ y = 2.20x^{0.333}, \quad 1 \leq x \leq 25 \]

where \( x \) is their age in years, and \( y \) is their height in feet.

a) How tall is the average 20 yr old male in Brazil

b) How old is the average male that is 5’10” from Brazil

Example 7: A slightly banked corner on the highway will safely handle speeds given by the equation:

\[ f(x) = \frac{24}{5} \sqrt{x} \]

where \( x \) represents the radius of the corner in feet, and \( f(x) \) represents the speed a car can travel safely in miles per hour.

- a) Evaluate \( f(30) \) and interpret.

- b) If the traffic in that school zone usually averages 25mph, what radius should the corner be?