Math142 Lecture Notes

2.3 - Instantaneous Rate of Change and the Derivative

Using the graph to the right of $f(x)$, find the slope of the secant line where $\Delta x = h$.

First find $f(x + h)$
Then find $f(x + h) - f(x)$
Then find $\frac{f(x + h) - f(x)}{h}$
which is the difference quotient:

$$\frac{f(x + h) - f(x)}{h} \text{ or } \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

What happens as $\Delta x$, or $h$, approaches zero?

To find the slope of the tangent at a point on a curve, find

$$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$
The U.S. imports from China for the years 1987 to 1996 were modeled by \( f(x) = 0.32x^2 + 1.64x + 3.98 \) where \( x \) represents the number of years since 1986 and \( f(x) \) represents the dollar value (in billions) of goods imported. Thus, the average rate of change from 1990 \((x = 4)\) to 1993 \((x = 7)\) is \( m_{sec} = 5.17 \), which means that over this three year period, U.S. imports from China increased, on average, 5.16 billion dollars per year. Now we take our analysis a step further and ask: At what rate were U.S. imports from China changing at the beginning of 1993?

**Instantaneous Rate of Change/Slope of Tangent Line/Derivative**

The **instantaneous rate of change** of a function, \( f \), at \( x \) is equal to the **slope of the line tangent** to the graph of \( f \) at \( x \) and is given by

\[
\text{Instantaneous rate of change} = m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

provided that the limit exists. Its units of measurement are units of \( f \) per unit of \( x \).

* \( f'(x) \) is the notation for the derivative of \( f \) and is read “f prime of x”

Example 1: Compute \( f'(2) \) given \( f(x) = x^2 - 3x \)

(b) Find the equation of the tangent to the curve \( f(x) = x^2 - 3x \) at the point \((2, -2)\).
Example 2: The total cost of producing $q$ recreational vehicles can be modeled by

$$C(q) = 100 + 60q + 3q^2$$

where $q$ is the number of vehicles produced and $C(q)$ is in hundreds of dollars.

(a) Find $C(5)$ and interpret.

(b) Find $C'(5)$ and interpret.

Example 3: Given $f(x) = -2x^2 + 3x$ find $f'(x)$ and use this to find $f'(3), f'(6),$ and $f'(1)$
Example 4: The U.S. exports of all merchandise to all countries can be modeled by

\[ f(x) = 5.18x^2 + 5.86x + 410.6, \quad 1 \leq x \leq 6 \]

where \( x \) is the number of years since 1990 and \( f(x) \) is the dollar value (in billions) of all U.S. exports to all countries.

(a) Determine \( f'(x) \)

(b) Use the result from (a) to compute \( f'(3) \), \( f'(4) \), and \( f'(5) \), and interpret each.

(c) Using your knowledge that the derivative gives the slope of a tangent line, does the result from part (b) indicate that the function is increasing or decreasing? Explain.