Math142 Lecture Notes
5.2 - Second Derivative and Graphs

First Derivative Second Derivative \( n^{th} \) derivative
\[
\begin{array}{lll}
  \frac{df}{dx} & \frac{d^2f}{dx^2} & \frac{d^n f}{dx^n} \\
  \frac{dy}{dx} & \frac{d^2y}{dx^2} & \frac{d^n y}{dx^n}
\end{array}
\]

Example 1: Find the first three derivatives for \( f(x) = \ln 3x \).

Example 2: Given \( f''(x) = 30x + 8 \), find where \( f''(x) = 0 \) then find the intervals where \( f'(x) \) is increasing and decreasing.

**Concavity**

On an open interval \((a, b)\) where \( f \) is differentiable:

1. If \( f'(x) \) is increasing \( (f''(x) > 0) \) then the graph of \( f(x) \) is concave up.
2. If \( f'(x) \) is decreasing \( (f''(x) < 0) \) then the graph of \( f(x) \) is concave down.
Example 2: Determine the intervals where the graph of \( f(x) = x^3 + 3x^2 - 4 \) is concave up and concave down.

**Second Derivative Test (determines inflection values and concavity)**

1. Find all inflection values. These are the values \( x = d \) where \( f''(d) = 0 \) or \( f''(d) \) is undefined.

2. Place these values on a number line and use the second derivative to generate a sign diagram.

3. The point \( (d, f(d)) \) is an inflection point if \( f''(x) \) changes sign at \( x = d \) and if \( x = d \) is in the domain of \( f(x) \).

Example 3: Find the inflection points of \( f(x) \) if \( f''(x) = (x + 2)(x - 1)x^2 \).
Example 4: The Chug-a-Mug Company has determined that its cost, in hundreds of dollars, for producing $x$ items of its best selling product is given by $C(x) = x^3 - 6x^2 + 15x$.

(a) Determine $C(5)$ and $MC(5)$ and interpret each.

(b) Determine the intervals where $MC(x)$ is increasing and where it is decreasing. Determine the relative minimum for the marginal cost function.

(c) Determine the inflection point for the graph of $C$. 