Math142 Lecture Notes
6.3 - Fundamental Theorem of Calculus

We know from section 6-2 that the area under the curve below was 9 square units.

In addition, we set up the definite integral \( \int_{0}^{3} \frac{2}{3}x + 2 \, dx \) to represent the area. Now let’s find the antiderivative \( F(x) \) of \( f(x) \) and then evaluate \( F(3) - F(0) \).

The Fundamental Theorem of Calculus
If \( f \) is a continuous function defined on a closed interval \([a, b]\) and \( F \) is an antiderivative of \( f \), then

\[
\int_{a}^{b} f(x) \, dx = F(x) \bigg|_{a}^{b} = F(b) - F(a)
\]

Example 1: Evaluate \( \int_{3}^{6} (4x - 2) \, dx \)

Example 2: Find the exact area under \( f(x) = x^2 - 2x + 3 \) on \([1, 3]\).
Values of the definite integral \( \int_a^b f(x) \, dx \) for various functions.

- If \( f(x) \) lies above the \( x \)-axis, \( \int_a^b f(x) \, dx > 0 \).
- If \( f(x) \) lies below the \( x \)-axis, \( \int_a^b f(x) \, dx < 0 \).

If a portion of the graph lies above the \( x \)-axis and another portion below the \( x \)-axis we can compute two different types of areas.

**Net Area**\(=\int_a^b f(x) \, dx = R_1 + R_2\)

**Gross Area**\(=\int_a^b f(x) \, dx = R_1 + |R_2|\)

Example 3: Given \( f(x) = |x| - 4 \)

(a) Rewrite \( f(x) \) as a piecewise function.

(b) Determine the \( x \)-intercepts of the graph.

(c) Find the net and gross areas of \( \int_0^9 f(x) \, dx \).
Example 4: The TinyTot Toy Company determines that the marginal cost for producing a new action figure is given by

\[ MC(x) = 4 - 0.02x, \quad 0 \leq x \leq 100 \]

where \( x \) is the number of toys made daily and \( MC(x) \) is the marginal cost measured in dollars per toy.

(a) Evaluate \( MC(30) \) and interpret.

(b) Evaluate \( \int_0^{30} MC(x) \) and interpret.
Example 5: Finding distance traveled given velocity functions

The velocity of an object can be modeled by \( v(t) = 12t + 40 \) where \( t \) is the time in seconds and \( v(t) \) is the velocity measured in \( \text{feet per second} \).

a. Find the distance traveled between 5 seconds and 25 seconds.

b. Evaluate \( v(10) \) and interpret.

Example 6: If the rate of change of sales of an item is given by \( S'(t) = 9t^2 + 18t \) where \( t \) is the number of weeks after an advertising campaign has begun, how many items were sold during the third week?