Math142 Lecture Notes
6.1 - Antiderivatives and Indefinite Integrals

Find all possible functions $F(x)$ whose derivative is $f(x) = 2x$.

- $F(x) = x^2$
- $F(x) = x^2 + 5$
- $F(x) = x^2 + \pi$
- $F(x) = x^2 + C$

**Definition: Antiderivative**

- If $F'(x) = f(x)$, then $F(x)$ is called an antiderivative of $f(x)$.
- If $C$ is any real number constant, then the general antiderivative of $f$ on an interval is $F(x) + C$ if $\frac{d}{dx}[F(x) + C] = f(x)$ for all $x$ in the interval. We use the notation
  \[ \int f(x)dx = F(x) + C \]
  to denote $F(x) + C$ as the general antiderivative of $f(x)$.

*This is read “The integral of $f(x)$ with respect to $x$ is $F(x) + C$”*

**Example 1:** Determine if the function $F$ is the general antiderivative of the function $f$.

(a) $F(x) = \frac{2}{3}x^{3/2} + 4x + C$; $f(x) = \sqrt{x} + 4$

(b) $F(x) = 2x^4 - x + C$; $f(x) = \frac{2}{3}x^3 - 1$
Power Rule for Integration
For any real number $n$, where $n \neq -1$, the indefinite integral of $x^n$ is

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

Example 2: Determine the following indefinite integrals.

(a) $\int x^9 \, dx$

(b) $\int \frac{1}{t^{11}} \, dt$

(c) $\int \sqrt{3} y^2 \, dy$

Other Rules for Integration

- **Constant Rule**
  
  If $k$ is any real number, then the indefinite integral of $k$ is $\int k \, dx = kx + C$.

- **Sum and Difference Rule**
  
  For integrable functions $f$ and $g$, $\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$.

- **Coefficient Rule**
  
  Given any real number coefficient $c$ and integrable function $f$,

  $$\int c \cdot f(x) \, dx = c \cdot \int f(x) \, dx.$$

Example 3: Find the following.

(a) $\int 0.5 \, x^7 \, dx$

(b) $\int (5x - 2) \, dx$
(c) \[ \int (4t^4 + 5t - 6) \, dt \]

(d) \[ \int \left( x^{5/2} - \frac{4}{x^3} - \sqrt{x} \right) \, dx \]

(e) \[ \int \frac{3y^2 - 2y}{6y} \, dy \]

Example 4: The Best Dressed Clothing Company finds that its marginal profit, \( MP \), is linear and has the form \( MP(q) = mq + b \), where \( m,b \) are constants. The company gets about $171 additional profit from producing the 101\textsuperscript{st} sport coat and $169 additional profit from producing the 151\textsuperscript{st} sport coat in each production run.

(a) Determine the marginal profit function \( MP \).

(b) Knowing that the company gets $11,300 profit from 150 sport coats, find the profit function \( P \).
Indefinite Integrals of Basic Functions

- **Power Rule**
  \[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \]

- **Rule for \( e^x \)**
  \[ \int e^x \, dx = e^x + C \]

- **Rule for Logarithmic Functions**
  \[ \int \frac{1}{x} \, dx = \ln |x| + C, \quad x \neq 0 \]

Example 5: Determine the following indefinite integrals.

(a) \[ \int \left( \frac{5}{x} - 8e^x \right) \, dx \]

(b) \[ \int \frac{x^4 - 5x^2}{x^5} \, dx \]

(c) \[ \int \left( \frac{4}{v} + \frac{v}{4} \right) \, dv \]

Example 6: If the marginal cost of producing \( x \) units is given by \( C'(x) = 0.9x^2 + 5x \) and the fixed cost is $5000, find the cost function \( C(x) \) and the cost of producing 25 units.