5.1 First Derivatives

a) Find the critical values of 
   \[ f(x) = -x^3 - 3x^2 + 45x - 5 \]

b) Find the critical values of \( g(x) = x + \ln x \)

c) Find where the function in part "a" is increasing or decreasing.

d) Find where the function in part "b" is increasing or decreasing.

e) Find any relative extrema from part a.

f) Find any relative extrema from part b.

5.2 Second Derivatives and Graphs

a) Find where the function is concave up or down for 
   \[ f(x) = -3x^3 + 5x^2 - 2x + 4 \]

b) Find where \( f(x) \) is increasing/decreasing, concave up/down, relative extremum, and points of inflection for each of the following functions:
   1) \( f(x) = 3x^2 - 2x - 5 \)
   2) \( g(x) = x + \frac{5}{x} \)

c) Sketch the graph described below:
   
   domain \( \mathbb{R} \)
   range \( \mathbb{R}, y \geq 5 \)
   continuous over \( \mathbb{R} \)
   \( f' < 0 \) \((-\infty, -3)\)
   \( f' > 0 \) \((-3, \infty)\)
   \( f'' > 0 \) \((-\infty, \infty)\)

5.3 Curve Sketching

a) Find the relative extrema of 
   \[ f(x) = x^4 + x^3 - 7x^2 - x + 6 \]

b) Graph and label relative extrema, inflection points, and asymptotes for the function:
   \[ f(x) = \frac{3x + 1}{5x - 2} \]

c) The Digital Pet Company has determined that its daily cost in dollars, for producing \( x \) virtual pets is given by \( C(x) = 150 + 3x + \frac{2x^2}{30} \), \( 0 \leq x \leq 200 \). Find the average cost function.

d) How many virtual pets should they make to minimize the average costs?

5.4 Optimizing Functions

a) Determine the absolute extrema:
   1) \( f(x) = x^3 + 4x^2 + x - 6 \) over \([-2, 0]\)
   2) \( f(x) = \frac{x}{x - 2} \) over \([3, 5]\)

b) From past records, the owner of the Sleep Cheap Motel has determined that when $x per day is charged to rent a room the daily profit, \( P(x) \), is given by 
   \[ P(x) = -x^2 + 92x - 180 \]

   \[ 40 \leq x \leq 60 \]

c) What should the owner charge to maximize profits?

d) Find the dimensions of a garden, \( x \) feet by \( y \) feet, where \( 10 \leq x \leq 100 \), so that the garden will have 1440 sq. ft of area, and be surrounded by a walkway that is 8ft wide on the north and south sides, and 5ft wide on the east and west sides, so that the Total Area (of garden and walkways) is a minimum.

e) The concentration of a certain medication in a patient’s bloodstream can be given by 
   \[ C(t) = \frac{5.3t}{t^2 + 4t + 5}, 0 \leq t \leq 8 \]  
   where \( C(t) \) is in milligrams per cubic centimeter and \( t \) is the number of hours after the medication has been administered.

   1) How many hours after the medication has been administered is the concentration at a maximum?

   2) What is the maximum concentration?

5.5 2nd Derivative Test & Optimization

a) Given \( f(x) = 2x^3 - 3x^2 - 72x - 150 \), find the absolute extrema on the interval \([-5,7.5]\)

b) Given \( f(x) = \frac{x - 1}{x^2} \), find the absolute extrema on the interval \((-\infty, \infty)\)

c) Find two numbers whose sum is 20 for which the sum of the squares is a minimum.

d) Find the dimensions of a closed box with a square base which has a volume of 27000 cubic inches with minimum surface area.

e) The local travel agency offers one week in Hawaii, for $850 if 20 sign up, and discounts of $20 per person for each additional person over the minimum of 15. If the 747 holds 300, what price should they charge to maximize revenue?

6.1 The Indefinite Integral

a) \( \int \sqrt{x^2} \, dx \)

b) \( \int (\sqrt{x} - \sqrt{x}) \, dx \)
c) \[ \int \frac{x^3 + x^5}{x^4} \, dx \]

d) \[ \int (e^x + \pi) \, dx \]

e) Find the cost function if the marginal cost in dollars is given by \(28x - 10e^x\) where \(x\) is the number of items sold and there are fixed costs of $50.

f) The population of Smallsville, OH is increasing at the rate \(2400e^{0.03t}\) where \(t\) is the number of years since 1950, when the population was 30,000. Find the population in 1985 according to this model.

6.2 Area and the Definite Integral

a) Write the definite integral to represent the shaded area:

\[ f(x) = x^3 + 1 \]

b) Find the approximate area under the curve \(f(x) = \frac{x^2}{\sqrt{x^2 + 8x + 7}}\) over the interval \([1, 13]\) using 100 rectangles.

c) Write a definite integral to represent the shaded area under the curve over the interval \([0, 5]\)

\[ f(x) = \begin{cases} 6 - x, & \text{if } x \leq 3 \\ x, & \text{if } x > 3 \end{cases} \]

d) Write a definite integral to represent the shaded area.

\[ f(x) = 9 - x^2 \]

e) Approximate the shaded area above using 100 rectangles by finding the left hand sum and the right hand sum.

f) Sketch and shade the region given by the definite integral \(\int_{0}^{13} \frac{4\sqrt{2x + 2}}{x} \, dx\).

g) Estimate the given integral with \(n=10:\)

\[ \int_{1}^{4} (5x^3 - 3x^2 + 9) \, dx \]

h) Estimate the given integral with \(n=10:\)

\[ \int_{1}^{4} (x \ln x) \, dx \]

6.3 The Fundamental Theorem of Calculus

a) \[ \int_{-2}^{2} (4 - x^2) \, dx \]

b) \[ \int_{1}^{5} e^{2x} \, dx \]

c) \[ \int_{0}^{1} xe^{x^2 + 1} \, dx \]

d) \[ \int_{0}^{1} \frac{1}{\sqrt{3x + 1}} \, dx \]

e) Refer to the figure below of \(F'(x)\). If \(F(0)=2\), what is \(F(4)\)? What is \(F(8)\)?

f) Suppose oil is being extracted at a rate \(.1e^{5t}\) where \(t\) is measured in years and \(P(t)\) in millions of barrels of oil. At this rate, how much oil will be extracted the 5th year?

g) The ScandiTrac Company determines that their marginal profit function for producing and selling a new economy model of cross-country ski machine at a mall is given by

\[ MP(x) = P''(x) = 0.3x^2 + 0.2x, \quad 0 \leq x \leq 30 \]

where \(x\) is the number of machines produced and sold and \(P'(x)\) is the marginal profit function measured in dollars per ski machine.

1) Knowing that $704 profit is made when 20 ski machines are sold, find the profit function \(P(x)\).

2) Evaluate \(\int_{10}^{20} P'(x) \, dx\) and interpret.

6.4 Substitution or ”Fix-It Method”

a) \[ \int 6(x + 1)(2x^2 + 4x - 5)^{\frac{3}{2}} \, dx \]

b) \[ \int 5x(3x^2 - 10)^{\frac{1}{2}} \, dx \]

c) \[ \int \frac{x}{\sqrt{x^2 - 1}} \, dx \]

d) \[ \int \frac{x^3}{\sqrt{x^4 - 4}} \, dx \]
c) \[ \int_{-5}^{0} \sqrt{2-x} \, dx \]

f) Find the exact area of the shaded region below:

[Image of a graph with a shaded region and the function \( f(x) = \frac{1}{\sqrt{2x+1}} \)]

6.5 Logarithmic and Exponential Functions

a) \[ \int \frac{1}{2x+3} \, dx \]

b) \[ \int \frac{6x^2 - 6}{x^3 - 3x + 4} \, dx \]

c) \[ \int \frac{1}{x \ln x} \, dx \]

d) \[ \int e^{2x} + e^{-2x} \, dx \]

e) \[ \int 4x \cdot 5x^2 + 1 \, dx \]

f) \[ \int (x + \frac{1}{x}) \, dx \]

Answer Key for Exam 3 Review

5.1

a. \( x=3,-5 \)

b. no critical values

c. increasing \((-5,3)\), decreasing \((-\infty,-5)\), \((3,\infty)\)

d. increasing \((0,\infty)\)

e. rel min at \( x=-5 \), rel max at \( x=3 \)

f. none

5.2

a. concave up \((-\infty, \frac{5}{3})\), concave down \((\frac{5}{3}, \infty)\)

b. (1) rel min at \( x = \frac{1}{3} \), inc \((\frac{1}{3}, \infty)\), dec \((-\infty, \frac{1}{3})\), concave up \((-\infty, \infty)\)

b. (2) rel min at \( x = \sqrt{5} \), rel max at \( x = -\sqrt{5} \), inc \((-\infty, -\sqrt{5})\), \((\sqrt{5}, \infty)\), dec \((-\sqrt{5}, 0)\), \((0, \sqrt{5})\), concave up \((0, \infty)\), concave down \((-\infty, 0)\)

5.3

a. rel min: \((-2.25, -12.95)\), \((1.57, -2.88)\)

rel max: \((0, 6)\)

b. rel extrema: none

inflec. points: none

asymptotes: \( x = \frac{2}{5} \), \( y = \frac{3}{5} \)

c. Avg Cost= \( \frac{150}{x} + 3 + \frac{4}{x^2} \)

d. 47 virtual pets

5.4

a. (1) absolute max at \( x = -2 \); absolute min at \( x = \frac{4}{3} + \frac{\sqrt{3}}{3} \)

a. (2) absolute max =3; absolute min = \( 1\frac{4}{3} \)

c. $46 /day

d. 30 x 48 feet

e. (1) \( t = 2.236 \) hrs; (2) .63mg/m^3

5.5

a. max -15; min -358

b. min: none; max \( \frac{1}{4} \)

c. \( x=10 \), \( y=10 \)

d. 30x30x30 inches

e. $630

6.1

a. \( \frac{4}{5} x^2 + C \)

b. \( \frac{2}{5} x^3 - \frac{4}{3} x^4 + C \)

c. \( \ln |x| + \frac{1}{2} x^2 + C \)

d. \( e^x + \pi x + C \)

e. \( C(x) = 14x^2 - 10e^x + 60 \)

f. approx 151,650 people

6.2

a. \( f_1^3 (x^2 + 1) \, dx \)

b. left=56.862; right = 58.044

c. \( f_0^3 (6 - x) \, dx + f_5^3 x \, dx \)

d. \( f_0^3 (9 - x^2) \, dx \)

e. left 18.135; right 17.865

g. left 243.8025; right 324.8025

h. left 6.519; right 8.183

6.3

a. \( 10.\overline{6} \)

b. exact: \( \frac{1}{6} e^{10} - \frac{1}{6} e^2 \)

approx: 11,009.538

c. exact: \( \frac{1}{2} e(e - 1) \)

approx: 2.335

d. \( \frac{2}{3} \)

e. 22; 32
f. 960,000 barrels of oil  

g. (1) $P = 0.1x^3 + 0.1x^2 - 136$

g. (2) $\$730; At a production level of 20 machines, the profit from the last 10 machines is $730.$

6.4

a. $\frac{3}{5} (2x^2 + 4x - 5)^{\frac{3}{2}} + C$

b. $\frac{5}{9} (3x^2 - 10)^{\frac{5}{2}} + C$

c. $\frac{2}{3} (x^2 - 1)^{\frac{2}{3}} + C$

d. $\frac{1}{2} (x^4 - 4)^{\frac{1}{2}} + C$

e. exact: $\frac{2}{3} (\sqrt{2401} - \sqrt{16})$

approx: 8.1530

f. $\frac{3}{4} \sqrt{81} - \frac{3}{4}$

6.5

a. $\frac{1}{2} \ln | 2x + 3 | + C$

b. $2 \ln | x^3 - 3x + 4 | + C$

c. $\ln | \ln | x || + C$

d. $\frac{1}{2} \ln | e^{2x} - e^{-2x} | + C$

e. $\frac{2 \cdot 5^{x^2 + 1}}{\ln 5} + C$

f. $\frac{x^2}{2} + \ln | x | + C$