Addition Algorithms

1. Concrete Model

2. Expanded Algorithm

\[ \begin{align*}
23 & \quad + 12 \\
+ & \quad \underline{5 \text{ (Add ones)}} \\
+ & \quad \underline{30 \text{ (Add tens)}} \\
35 & \quad \\
\end{align*} \]

3. Standard Algorithm

\[ \begin{align*}
23 & \quad + 12 \\
+ & \quad \underline{35} \\
\end{align*} \]

For combinations larger than nine, in elementary grades, use the words “regroup” or “trade” instead of “carry” to explain the process.

4. \[ \begin{align*}
46 & \quad + 35 \\
+ & \quad \underline{11 \text{ (Add ones)}} \\
+ & \quad \underline{70 \text{ (Add tens)}} \\
81 & \\
\end{align*} \]

Adding in Other Bases

5. Get a “place-value mat” and some pennies.
6. On the mat, place the pennies to model \( 23_{\text{four}} + 33_{\text{four}} \).
7. To add these numbers we must regroup because there are too many single units for our base. So \( 23_{\text{four}} + 33_{\text{four}} = \)

Problem 1: Use the “place value mat” and pennies to find the following sums.

a) \( 12_{\text{four}} + 22_{\text{four}} = \)

b) \( 33_{\text{four}} + 22_{\text{four}} = \)

c) \( 21_{\text{four}} + 33_{\text{four}} = \)

d) \( 42_{\text{five}} + 14_{\text{five}} = \)

e) \( 13_{\text{five}} + 24_{\text{five}} = \)

f) \( 41_{\text{five}} + 40_{\text{five}} = \)

Problem 2: Use the standard algorithm to find the following sums.

a) \( 12_{\text{four}} \quad + 22_{\text{four}} \)

b) \( 33_{\text{four}} \quad + 22_{\text{four}} \)

c) \( 21_{\text{four}} \quad + 33_{\text{four}} \)
Problem 3: Use the expanded algorithm to find the following sums.

\[
\begin{align*}
\text{a) } & \quad 12_{\text{four}} + 22_{\text{four}} = \\
\text{b) } & \quad 33_{\text{four}} + 22_{\text{four}} = \\
\text{c) } & \quad 21_{\text{four}} + 33_{\text{four}} = \\
\text{d) } & \quad 42_{\text{five}} + 14_{\text{five}} = \\
\text{e) } & \quad 13_{\text{five}} + 24_{\text{five}} = \\
\text{f) } & \quad 41_{\text{five}} + 40_{\text{five}} = \\
\end{align*}
\]

\[\star\text{Subtracting in Other Bases}\]

1. Get a “place-value mat” and some pennies. We will use the mat, to model \(21_{\text{four}} - 13_{\text{four}}\). To do this lay out enough pennies to represent \(21_{\text{four}}\).
2. To subtract these numbers we must regroup because we can not take 3 units from 1 unit. So we take a stack of four pennies and give them to the ones place.
3. Now take away 1 stack of 4 and 3 single pennies.
4. So \(21_{\text{four}} - 13_{\text{four}} = \)

Problem 4: Use the “place value mat” and pennies to find the following differences.

\[
\begin{align*}
\text{a) } & \quad 31_{\text{four}} - 12_{\text{four}} = \\
\text{b) } & \quad 20_{\text{four}} - 3_{\text{four}} = \\
\text{c) } & \quad 42_{\text{five}} - 14_{\text{five}} = \\
\text{d) } & \quad 31_{\text{five}} - 22_{\text{five}} = \\
\end{align*}
\]

Problem 5: Use the standard algorithm to find the following differences.

\[
\begin{align*}
\text{a) } & \quad 31_{\text{four}} - 12_{\text{four}} = \\
\text{b) } & \quad 20_{\text{four}} - 3_{\text{four}} = \\
\text{c) } & \quad 42_{\text{five}} - 14_{\text{five}} = \\
\text{d) } & \quad 31_{\text{five}} - 22_{\text{five}} = \\
\end{align*}
\]
Other Algorithms for Adding Whole Numbers (base 10)

1) Left-to-Right Algorithm for Addition:

\[
\begin{array}{c}
587 \\
+ 632 \\
\end{array}
\]

2) Lattice Algorithm for Addition

\[
\begin{array}{c}
5 \ 4 \ 3 \ 7 \\
+ 2 \ 8 \ 7 \ 5 \\
\hline
9 \ 2 \ 4 \ 0
\end{array}
\]

3) Scratch Algorithm for Addition

\[
\begin{array}{c}
94 \\
57 \\
31 \\
+ 24 \\
\hline
126
\end{array}
\]

4) Equal Addends Algorithm