Math 365 Lecture Notes
Section 3.3 – Algorithms for Whole-Number Multiplication and Division

★Multiplying by Whole Numbers

1. Using the distributive property:
   \[ 4 \times 12 = \]
   \[ 4 \times (10 + 2) = \]
   \[ 4 \times 10 + 4 \times 2 = 40 + 8 = 48 \]

2. Multiplying with Zeros
   \[ 5 \times 3,000 = 5 \times (3 \times 10^3) = \]
   \[ = (5 \times 3) \times 10^3 = \]
   \[ = 15 \times 10^3 = 15,000 \]

3. Is the Answer Reasonable?
   \[ 14 \leftrightarrow 23 = ??? \]
   \[ 10 \times 20 = 200 \]
   \[ 14 \leftrightarrow 23 = ??? \]
   \[ 20 \times 30 = 600 \]

4. Concrete Models

5. Lattice Multiplication

★Multiplying by a Single Digit in Other Bases

6. Get a “place-value mat” and some pennies.

7. On the mat, place the pennies to model \( 12_{\text{four}} \times 3_{\text{four}} \).

8. To multiply these numbers we must regroup because there are too many single units for our base. So \( 12_{\text{four}} \times 3_{\text{four}} = \)

Problem 1: Use the “place value mat” and pennies to find the following products.
   a) \( 13_{\text{four}} \times 3_{\text{four}} = \)
   d) \( 42_{\text{five}} \times 2_{\text{five}} = \)
Problem 2: Use the standard algorithm to find the following products.

a) \[ 13_{four} \times 3_{four} = \]

b) \[ 33_{four} \times 2_{four} = \]

c) \[ 21_{four} \times 3_{four} = \]

e) \[ 13_{five} \times 3_{five} = \]

f) \[ 41_{five} \times 4_{five} = \]

Problem 3: Use the expanded algorithm to find the following products.

a) \[ 42_{five} \times 2_{five} = \]

b) \[ 13_{five} \times 3_{five} = \]

c) \[ 41_{five} \times 4_{five} = \]

Multiplying by Two Digits in Other Bases

Problem 4: Use the standard algorithm to find the following products.

a) \[ 31_{four} \times 12_{four} = \]

b) \[ 42_{five} \times 14_{five} = \]

c) \[ 31_{five} \times 44_{five} = \]

Building a Multiplication Table

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Multiply \( 231_{five} \times 122_{five} \)
**Short Division**

**Dividing in by a Single Digit in Other Bases**

Problem 5: Use the standard division algorithm to find the following quotients.

\[
\begin{align*}
\text{a)} & \quad 4_{\text{five}} \overline{1442_{\text{five}}} \\
\text{b)} & \quad 3_{\text{five}} \overline{20312_{\text{five}}} 
\end{align*}
\]