Math 365 Lecture Notes
Section 4.2 – Multiplication and Division of Integers

☆ Integer Multiplication

Signed Counter Model  Also called charged-field model or Chip Model

$3 \times -2 \rightarrow$ Use the commutative property to rewrite the problem as $-2 \times 3$ and then interpret the problem as “remove 2 groups of 3 positive counters”.

$-2 \times 4 \rightarrow$ Interpret the problem as “remove 2 groups of 4 positive counters”.

$-2 \times -3 \rightarrow$ Interpret the problem as “remove 2 groups of 3 negative counters”.

Pattern Model: Establish a pattern with previously known facts and then continue the pattern to learn new facts.

$3 \times -5$  $-2 \times -3$

Number Line Model:
1. Traveling to the left (west) means moving in the negative direction, and traveling to the right (east) means moving in the positive direction.
2. Time in the future is denoted by a positive value, and time in the past is denoted by a negative value.

$3 \times 5$

$3 \times -5$

$-5 \times 3$

$-3 \times 5$
**Integer Division**

**Definition of Integer Division:**

**Problem 1:** Use the definition of integer division to evaluate each of the following, if possible.

a) \( 24 \div 4 \)

b) \( -28 \div 7 \)

c) \( -30 \div -6 \)

**Properties:**

The set of integers \( I \) satisfies the following properties of multiplication for all integers \( a, b, c \in I \).

**Closure property of multiplication of integers** –

**Commutative property of multiplication of integers** –

**Associative property of multiplication of integers** –

**Multiplicative identity property** –

**Distributive properties of multiplication over addition for integers** –

**Distributive property of multiplication over subtraction for integers** –

**Zero multiplication property of integers** –

**Other properties:** For all integers \( a \) and \( b \)

1) \((-1)a = \)

2) \((-a)b = \)

3) \((-a)(-b) = \)

**Other Important Concepts**

**Definition of Less Than for Integers:** For any integers \( a \) and \( b \), \( a \) is less than \( b \) \( (a < b) \), if and only if there exists a positive integer \( k \) such that

**Difference of Squares Formula:**
Less Than

\[ a < b \iff \exists k > 0 \text{ such that } a + k = b \]

Property

\[ a < b \iff b - a > 0 \]