Math 365 Lecture Notes
Section 4.4 – Prime and Composite Numbers

Definitions/Theorems
1) Prime Number –

2) Composite Number –

3) Factorization –

4) Prime Factorization –

5) Fundamental Theorem of Arithmetic –

Finding Prime Factorizations
1) Factor Trees –

2) using repeated division, and the list of prime numbers (2, 3, 5, 7, 11, 13, 17, …)

Problem 1: Find the prime factorization of 380.

Number of Divisors
Exploration:
1) Find the prime factorization of 30.

2) Find all factors of 30.
3) What is the relationship between the prime factorization of 30 and the number of factors of 30?

4) Does the relationship found in question 3) above hold true for 32?

5) Describe a process for finding the number of divisors of any natural number.

**Problem 2:** Find the number of divisors for \(16,200 = 2^3 \times 3^1 \times 5^2\).

★ Determining if a Number is Prime

**Theorems:**

1) 

2) 

3) 

**Problem 3:** Explain the process to determine if 247 is prime or composite.

**Problem 4:** On Monday the local toy store sold \(x\) stuffed dogs, and on Friday they sold \(y\) stuffed dogs. If they collected $5665 on Monday, and $8239 on Friday, how many did they sell on Monday?
**Sieve of Eratosthenes**

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**Large Primes**

a. \( n^2 - n + 41 \), \( n \in \mathbb{W}, n < 41 \) always produces a prime number

b. Mersenne primes found in 1998 by Cal State student

\[ 2^{3021377} - 1 \]
\[ 2^{6972593} - 1 \]
\[ 2^{13466917} - 1 \]

c. Sophie Germain primes \( p \) an odd prime, \( 2p+1 \) is also prime

\( p = 3 \) \( \rightarrow \) 3, 7
\( p = 5 \) \( \rightarrow \) 5, 11
\( p = 11 \) \( \rightarrow \) 11, 23

**Practice**

Are the following numbers prime or composite?

a. 12,345
b. 8,142
c. 524,331
d. 352,253
e. \((7)(5)(11)(17)+1\)
Problem Solving

The total sales on the first day were $2323 and on the second day were $2461. Only stuffed rabbits were sold. How many were sold each day?