Week-In-Review 7 on 4.2, 4.3, and Chapter 3 – 4.3

1. Change $111011_{\text{two}}$ to base six.

2. Prove why the product of two negative integers is a positive integer.

3. Use any two models discussed to explain $-6 \cdot 2$ to a child.

4. $63T_{\text{twelve}} - 5EE_{\text{twelve}} = \quad$ 

5. Order $-5 \cdot 25^{39}$, $-4 \cdot 5^{79}$, and $-2 \cdot 125^{27}$.

6. How many hours, minutes, and seconds are there in 7415 seconds? Explain how this problem relates to a base numeration system.

7. Use divisibility rules to determine whether the number 12,789 is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

8. Indicate by circling the appropriate letter whether the following properties apply to Addition, Subtraction, Multiplication, and Division over the set of integers.

   \begin{tabular}{l|c|c|c|c}
   \hline
   & A & S & M & D \\
   \hline
   Closed & A & S & M & D \\
   Commutative & A & S & M & D \\
   Identity Element & A & S & M & D \\
   \hline
   \end{tabular}

9. Use any two models discussed to explain $3 - 2$ to a child.

10. Find the first two terms of an arithmetic sequence in which the fifth term is $-13$ and the $100^{\text{th}}$ term is $-488$.

11. Write the first 15 counting numbers in base three.

12. Fill in the blanks with the appropriate word (factor, multiple, divisor).

   Given $18 | 54$. Eighteen is a \underline{______________} or \underline{______________} of 54. Fifty four is a \underline{______________} of 18.

13. Define place value.

14. The first movie to gross over $100,000,000$ was released in MCMXXXIX, which is what year?

15. Use mental math to calculate

   a. $540 ÷ 4$
   b. $74 - 38$
   c. $69 + 26$
   d. $47 \cdot 3$
16. \(462_{\text{seven}} + 555_{\text{seven}} = \)

17. Prove \((x + y)(x + y) = x^2 + 2xy + y^2\). Show all steps.

18. Use an algorithm to find 47 + 25.

19. If \(2021_{\text{three}} = 75_b\), then what does \(b\) equal?

20. Without performing the division, determine the following and explain your reasoning.
   a. Is 185,074 divisible by 37?
   b. Is 4,500,076 divisible by 19?
   c. Is \((5)(11)(19) + 3\) divisible by 5? Explain.

21. What is the counting number immediately before \(\text{EE0}_{\text{twelve}}\)?

22. Find all integers \(x\), if there are any, such that the following are true.
   a. \(-x\) is nonnegative
   b. \(-x - 5\) is negative
   c. \(-|x| = 8\)
   d. \(-|x|\) is negative

23. \(11540_{\text{eight}} \div 76_{\text{eight}} = \)

24. Of the four basic operations on integers, which have the associative property? If the operation on integers has the associative property, write, “yes” and give an example. If the associative property does not hold, write, “no” and give a counterexample.

25. Write 142\(_{\text{ten}}\) in
   a. Egyptian numeration
   b. Babylonian numeration
   c. Mayan numeration
   d. Roman numeration

26. Use an algorithm to find 53 – 27.

27. Compare and contrast the Egyptian, Babylonian, Mayan, Roman and Hindu-Arabic numeration systems.

28. Using the definition of \(a^n\), where \(n\) is any natural number, show the intermediate step.
   \[3^7 = \_\_\_\_\_\_\_\_\_\_\_ = 2187\]

29. Prove or disprove: If \(x\) is an integer, then \(-x^2 < 0\).
30. 11 | 44 if, and only if there is a unique integer k such that
________________________. In this case k = ______________________.

31. Bananas normally sell for 13 cents each. They go on sale for 3 for 36 cents. How many cents are saved if you purchase three dozen bananas while they are on sale. Write an expression that describes this problem (before any simplification) and then simplify it to find the answer.

32. Using the definition of less than, show –8 < –5.

33. Use an algorithm to compute \( 762 \div 6 \).

34. Factor completely \( 81x^2 - (x - 4y)^2 \).

35. Model \(-5 + 6\).

36. In the 2000 Olympics, the gold medal women’s long jumper jumped 7 yards, 2 feet, 1 inch. The gold medal men’s long jumper jumped 9 yards, 1 foot, 1 inch. How far did the two gold medalists jump combined? (Do not simply convert to inches.)

37. Prove or disprove: If a number is divisible by 12, then it is divisible by 6 and 2.

38. \(-|-6| - 5^2 + (-1)^3 + 48 \div (-4) \cdot 3 + (-4)^3 - (4 + 18) + 0 \cdot 9 =

39. Model \((-3)(-2)\).

40. What is the additive inverse of \(6x - 4y\)? Use the definition of additive inverse, show all steps, and then simplify your answer.

41. Is 42885 divisible by 2, 3, 4, 5, 6, 8, or 9?

42. Justify each step.

\[
29 - 7 = (2 \cdot 10 + 9 \cdot 1) - 7 \cdot 1 \\
= (2 \cdot 10 + 9 \cdot 1) + -7 \cdot 1 \\
= 2 \cdot 10 + (9 \cdot 1 + -7 \cdot 1) \\
= 2 \cdot 10 + (9 + -7) \cdot 1 \\
= 2 \cdot 10 + 2 \cdot 1 \\
= 22
\]
43. Why is \( \frac{-9}{0} \) undefined?

44. \( 438 \cdot 75 = \)

45. Justify this alternative proof that demonstrates a negative times a negative is a positive number. Let \( a, b \in R \).
   \begin{align*}
   \text{a. } & (-a) * 0 = 0 \\
   \text{b. } & (-a)[b + (-b)] = 0 \\
   \text{c. } & (-a)b + (-a)(-b) = 0 \\
   \text{d. } & -(ab) + (-a)(-b) = 0 \\
   \text{e. } & (-a)(-b) = ab
   \end{align*}

46. Let \( x, y \) and \( z \) be integers.

   True or False   a. If \( x \mid 24 \), then \( x \mid (5 \cdot 24) \).

   True or False   b. If \( 7 \mid y \) and \( 7 \mid z \), then \( 7 \mid (z - y) \).

   True or False   c. \( 13 \mid 1 \)

   True or False   d. If \( y \mid (x + z) \) then \( y \mid (y + z) \).

   True or False   e. \( 9 \mid 0 \)

47. If you buy a DVD player for $285, surround sound for $158, two boxes of RW DVDs for $29 per box, and 3 DVD movies for $24 each, estimate your monthly payment if you will pay off your purchase in one year?

48. Prove or disprove: If \( a \) divides \( b \), and \( a \) divides \( c \), then \( a \) divides \( b + c \).

49. Use an algorithm to compute \( 58 \cdot 67 \).

50. Explain all steps when performing the standard algorithm on \( 103 - 67 \).