Exam # 1A
Math 141.502
Fall 2000
(Sections 1.1-1.5, 2.1-2.6, and 3.1)

Name: ____________________________ Student ID: ____________________________

Signature: _______________________

Calculators, pencils and pens are permitted. Additional blank paper will be provided. No other materials may be used on the exam.

There are 5 multiple choice problems, worth 5 points each. There are 5 work out problems. There is one extra credit problem worth a maximum of 3 points.

You will receive 2 extra credit points if you complete the survey on the following page.

Note: Partial credit will be awarded, according to completeness of work. Write the answers, in order, on the blank pages provided. Clearly indicate the solution to each problem.
**MULTIPLE CHOICE**

1. If the distance between (0,1) and (3,k) is 5, what is k?

   A) 3  B) 0  C) 5  D) -1  E) none of these

   **Solution** The distance formula gives
   \[
   (3 - 0)^2 + (k - 1)^2 = 25 \\
   9 + (k - 1)^2 = 25 \\
   (k - 1)^2 = 16 \\
   k - 1 = \pm 4 \\
   k = 5, -3
   \]

   So the correct answer is C.

2. What is the equation of a circle, of radius 5, centered on (1,-1)?

   A) \((x - 1)^2 + (y + 1)^2 = 25\)  B) \((x + 1)^2 + (y - 1)^2 = 5\)
   C) \((x + 1)^2 + (y - 1)^2 = 25\)  D) \((x - 1)^2 + (y + 1)^2 = 5\)  E) none of these

   **Solution** From the formula for a circle,
   \[
   (x - 1)^2 + (y - (-1))^2 = 5^2 \\
   (x - 1)^2 + (y + 1)^2 = 25
   \]

   So the correct answer is A.

3. Find the length of the triangle formed by the three points P=(0,1), Q=(1,2), R=(-2,2)

   A) \(\sqrt{2} + 3 + \sqrt{5}\)  B) \(\sqrt{2} + 3 + \sqrt{5}\)
   C) \(\sqrt{2} + \sqrt{3} + \sqrt{5}\)  D) \(2 + 3^2 + 5\)  E) none of these

   **Solution** The length of PQ is \(\sqrt{2}\), QR is 3, RP is \(\sqrt{5}\), so the correct answer is B.

4. The equation of the line passing through (1,3) and (-2,-1) is

   A) \(3y = 4x + 5\)  B) \(4x + 3y + 5 = 0\)
   C) \(y = \frac{4}{3}x + 5\)  D) \(y = -\frac{1}{3}x + \frac{5}{3}\)  E) none of these

   **Solution** The slope of the line through these points is \(\frac{3 - (-1)}{1 - (-2)} = \frac{4}{3}\), so the equation is given by
   \[
   \frac{y - 3}{x - 1} = \frac{4}{3} \\
   3(y - 3) = 4(x - 1) \\
   3y - 9 = 4x - 4 \\
   3y = 4x + 5
   \]

   So the correct answer is A.
5. A car worth $25,000.00 initially is depreciated linearly over 7 years. If it has a scrap value (at the end of 7 years) of $2,000.00, what is it worth after 4 years?

A) $10,714  B) $11,857  C) $12,500  D) $17,000  E) none of these

**Solution**  The slope of the depreciation curve is given by

\[
\frac{25 - 2}{0 - 7} = -\frac{23}{7}
\]

The intercept is (0,25), so the equation of the line is

\[y = 25 - \frac{23}{7}x\]

When \(x = 4\), \(y = 11,857.14\), so the correct answer is B.

**WORKOUT**

1. [20 pts] Based on a survey of retailers, it is estimated that 1000 people will buy a color TV priced at $195.00. For every $10.00 the price drops, and additional 50 people will buy it. Suppliers will not ship the TV if the price drops below $150.00, but for every $10.00 increase in price, they will ship an additional 200 units.

   (a) Find the supply function \(S(x)\)

   (b) Find the demand function \(D(x)\)

   (c) Find the market equilibrium

**Solution**  The slope of the demand curve is given by

\[
\frac{195 - 185}{1000 - 1050} = \frac{10}{-50} = -\frac{1}{5}
\]

so the equation of the demand curve is given by

\[
\frac{p - 185}{x - 1050} = -\frac{1}{5}
\]

so

\[p - 185 = -\frac{1}{5}(x - 1050)\]

\[p = -\frac{1}{5}x + 395\]

The slope of the supply curve is given by

\[
\frac{160 - 150}{200 - 0} = \frac{10}{200} = -\frac{1}{20}
\]

so the equation of the supply curve is given by

\[
\frac{p - 150}{x - 0} = \frac{1}{20}
\]

so

\[p - 150 = \frac{1}{20}(x)\]

\[p = \frac{1}{20}x + 150\]
A sketch of the supply and demand curves, along with their intersection, is given below.

2. [15 pts] Use reverse shading to indicate the region defined by

\[ x > 0 \\
\]
\[ y > 0 \\
\]
\[ x \neq y \]
\[ y < 3 \\
\]
\[ y > x \]

Solution
3. [15 pts] Given the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

(a) What is linear equation which best fits this data?
(b) If x=10, what is the predicted value of y?
(c) If x=50, what is the predicted value of y?

**Solution**

(a) Entering the x values into \( L_1 \) and the y-values into \( L_2 \), then using the TI83 function LinReg(ax+b), we get \( a = 0.4955 \) and \( b = 4.2381 \).

(b) When x=10, the regression line predicts \( y = 0.4955 \times 10 + 4.2381 = 9.1931 \)

(c) When x=50, the regression line predicts \( y = 0.4955 \times 50 + 4.2381 = 29.0131 \)

4. [15 pts] Given the augmented matrix below, use the TI83 program ROWOPS to fill in the missing entries:

\[
\begin{bmatrix}
1 & 3 & 2 & 4 \\
2 & 0 & 0 & 5 \\
3 & -3 & 2 & 6
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & 3 & 2 & 4 \\
0 & X & X & X \\
0 & X & X & X
\end{bmatrix}
\]

**Solution** Applying the TI83 program "ROWOPS", or doing the row reduction by hand, we get

\[
\begin{bmatrix}
1 & 3 & 2 & 4 \\
0 & -6 & -4 & -3 \\
0 & -12 & -4 & -6
\end{bmatrix}
\]

5. [10 pts] Given the system

\[
\begin{align*}
2x_1 - x_2 - x_3 &= 0 \\
3x_1 + 2x_2 + x_3 &= 7 \\
x_1 + 2x_2 + 2x_3 &= 5
\end{align*}
\]

(a) write the system as a matrix equation \([A]x = b\).
(b) solve the system, using \([A]^{-1}\).

**Solution**

(a) In terms of matrices,

\[
\begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & 1 \\
1 & 2 & 2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0 \\
7 \\
5
\end{bmatrix}
\]

(b) Using the inverse of \([A]\),

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & 1 \\
1 & 2 & 2
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
7 \\
5
\end{bmatrix} = \begin{bmatrix}
1 \\
2 \\
0
\end{bmatrix}
\]
6. [3 pts EXTRA CREDIT] If

\[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
\end{bmatrix}
\begin{bmatrix}
x & y \\
z & w \\
\end{bmatrix}
= 
\begin{bmatrix}
1 & 1 \\
1 & 1 \\
\end{bmatrix}
\]

What are the values of \{x, y, z, w\}? \textbf{You must show your work!}

\textbf{Solution} The simplest way to solve this is to just apply the inverse of the left matrix

\[
\begin{bmatrix}
x & y \\
z & w \\
\end{bmatrix}
= 
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
1 & 1 \\
1 & 1 \\
\end{bmatrix}
= 
\begin{bmatrix}
-1 & -1 \\
1 & 1 \\
\end{bmatrix}
\]