1. (30) Suppose that M-Corp stock is currently selling for $20 a share. That, $S_u = 22$, $S_d = 19$, and $e^r = 1.01$. A broker decides to offer a European call option with expiration in one year, and a strike price of $20.50.

(a) What is the numerical value of the delta hedge ratio?

$$a = \frac{1.50 - 0}{22 - 19} = \frac{1}{2}$$

(b) Determine the arbitrage free price of this option (show at least three place accuracy).

$$V_0 = \frac{1}{2} 20 + \frac{1}{1.01} \left( 1.5 - \frac{1}{2} 22 \right) = 0.594$$

(c) A broker sells 50,000 options, and receives 6 cents more than the arbitrage free price for each option sold. The broker decides to hedge his position. How many shares of stock should be bought or sold, and what is the brokers profit when the option expires?

A hedge position for the broker is given by the expression

$$-50,000 (V - aS) = -50,000V + 25,000S$$

Thus, the broker should buy 25,000 shares of stock. The cost of setting up this portfolio is

$$25,000 (20) - 50,000 (0.634) = 467,297.03$$

After one year the broker will owe

$$467,297.03 (1.01) = 471,970.00$$

The profit, assuming that the stock depreciates is calculated as follows.

$$25,000(19) - 471,970 = 3,030.00$$

The same value is attained if the stock appreciates in value to 22.00
2. (30) The table below contains closing prices for stock in X-Corp.

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb. 4</th>
<th>Feb. 5</th>
<th>Feb. 6</th>
<th>Feb. 7</th>
<th>Feb. 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>52.90</td>
<td>53.52</td>
<td>53.66</td>
<td>54</td>
<td>54.54</td>
</tr>
</tbody>
</table>

(a) Compute the drift parameter $\mu$ and volatility $\sigma$ of X-Corp stock from this data. The ratios of the stock prices are

<table>
<thead>
<tr>
<th>Date</th>
<th>$S_1/S_0$</th>
<th>$S_2/S_1$</th>
<th>$S_3/S_2$</th>
<th>$S_4/S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>1.0117</td>
<td>1.0026</td>
<td>1.0063</td>
<td>1.01</td>
</tr>
</tbody>
</table>

The drift $\mu$ and volatility $\sigma$ equal

\[
\mu = \frac{1}{4} (1.0117 + 1.0026 + 1.0063 + 1.01) - 1 \frac{1}{365} = 2.798
\]

\[
S^2 = 1/3 \sum_{i=1}^{4} \left( \frac{S_i}{S_0} - m \right)^2, \text{ where } m = \frac{1}{4} (1.0117 + 1.0026 + 1.0063 + 1.01) = 0.000016
\]

\[
\sigma = \frac{S}{\sqrt{dt}} = 0.0773
\]

(b) Determine the $u$ and $d$ factors for a binomial model of stock prices for X-Corp with the time step being one day. Use one year as the basic unit of time.

\[
u = 1 + \mu dt + \sigma \sqrt{dt} = 1.0117
\]

\[
d = 1 + \mu dt - \sigma \sqrt{dt} = 1.0036, \text{ with } dt = \frac{1}{365}
\]

(c) Determine the $u$ and $d$ factors for a binomial model of stock prices for X-Corp with the time step being one month. Use one year as the basic unit of time.

\[
u = 1 + \mu dt + \sigma \sqrt{dt} = 1.255
\]

\[
d = 1 + \mu dt - \sigma \sqrt{dt} = 1.2109, \text{ with } dt = \frac{1}{12}
\]
3. (30) The following is a tree of stock prices. Determine the price tree of an American put option with a strike price of $26. Assume the annual interest rate equals $e^{r} = 1.02$. Each time step in the tree below represents one month.

   The value of the put option at expiration is given by max($26 - S, 0$). At earlier time nodes the option value is given by the larger of the chaining value or the exercise value. The option tree is shown below:

4. (10) A one month European call option is currently selling for $2.00. The strike price of the option is $40$, and the current stock price is $43$. The monthly interest rate is 0.5%. This presents an opportunity for arbitrage. Explain why and how much free profit can be made.

   The option is clearly undervalued. It’s price should be at least $3.00$. So, buy an option and short one share of stock. This generates an income of $41.00$ and after one month it grows with interest to

   $$41(1.005) = 41.205$$

   At expiration either the stock is worth 40 dollars or more or its value is less than 40. If the stock is worth 40 or more, then exercise the call option which you own. This gives you a share of stock with which you close out the short position. Your profit is then

   $$41.205 - 40 = 1.205$$

   If the value of the stock is less than 40, the option expires worthless. Buy a share of stock for $S < 40$ to close out the short position. The profit is then

   $$41.205 - S > 1.205.$$