The following exam consists of 11 problems worth a total of 100 points. There are 6 multiple choice questions worth 5 points each. There are 5 work-out problems. Partial credit will be awarded on the work-out problems, according to completeness of work. Note: You may wish to do the work out problems first, and then do the multiple-choice...

Write the answers to each problem down on the blank sheets provided. Indicate clearly your answer and any work you wish to be considered for partial credit.

You may begin the exam when the instructor indicates.
1. The minimum value of the function $P = 4x + 3y$ subject to the following constraints

\[
\begin{align*}
x + y & \geq 1 \\
x + 2y & \leq 3 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

is given by

**Solution:**

\[
\begin{align*}
(0,1), (1,0), (3,0), \text{ and } (0,3/2).
\end{align*}
\]

The vertices are (0,1),(1,0),(3,0), and (0,3/2). The values of the objective function are 3, 4, 12 and 9/2 respectively.

The **minimum** value of $P$ is therefore 3, which occurs at the point (0,1).

2. If $x \in A \cap (B \cup C)$ then which of the following is true

**Solution:**

If you graph the set as a Venn diagram

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You can see that the set consists of elements which are in $A$, and either in $B$ or $C$.

3. **True or False.** If $A$ and $B$ are disjoint, and if $B$ and $C$ are disjoint, then $A$ and $C$ are also disjoint!

**Solution:** False. See the picture below.
4. In bridge, each of the four players receives 13 cards. How many such hands are there?

**Solution:** There are \( C(52,13) \) ways to give 13 cards to the first player, \( C(39,13) \) possibilities for the second, \( C(26,13) \) for the third, and \( C(13,13) \) for the fourth. If we don’t distinguish between the players, that gives \( C(52,13)C(39,13)C(26,13)C(13,13) \) as the number of possible hands.

5. At SBISA Dining Hall, 300 students ate breakfast, 500 students at lunch and 400 students ate dinner. 200 students ate both breakfast and lunch, 100 students ate both breakfast and dinner, and 250 students ate both lunch and dinner. Furthermore, 50 students ate all three meals. How many students ate only breakfast?

**Solution:**

The number of students who **ate only breakfast** are 50.

6. Let \( U \) be the set \( \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \), and \( A, B, C \) be the sets \( \{1, 2, 3\} \), \( \{3, 4, 5\} \), \( \{2, 4, 6\} \) respectively. The set \((A \cap B) \cup C\) is given by

**Solution:**

\((A \cap B) = \{3\}\) and therefore \((A \cap B) \cup C = \{2, 3, 4, 6\}\)
7. Maximize the function \( P = x + y \) subject to the following inequalities

\[
\begin{align*}
x + y & \geq 1 \\
y & \leq 3 \\
y & \geq x \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

a) Graph the feasible region.

**Solution:** See below

![Graph of feasible region](image)

b) Find (and label) all the vertices.

**Solution:** (See above)

c) Find the maximum of \( P \) by the method of corners

**Solution:** The values of \( P \) at the vertices \((0,1), (1/2,1/2), (3,3) \) and \((0,3)\) are 1, 1, 6, and 3 respectively. The maximum of \( P \) is 6 and occurs when \( x=3 \) and \( y=3 \).

8. A student casts two die.

a) List the events where the two die differ by at least two.

**Solution:** \{\((1,3), (1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,5), (3,6), (4,6), (3,1), (4,1), (5,1), (6,1), (4,2), (5,2), (6,2), (5,3), (6,3), (6,4)\}\). Note, there are 20 events rather than 10, since \((1,3)\) and \((3,1)\) are both events.

b) What is the probability that the die differ by two or more?

**Solution:** The probability is given by the ratio 20/36 = 5/9.

9. A seven character password is constructed of uppercase letters, lower case letters, and the numbers (0-9).

a) If there are no other restrictions on the password, how many passwords are there?

**Solution:** Since there are 62=26+26+10 different possibilities for each character, with no restrictions we have \( 62^7 \) different combinations.

b) If the password must begin with a letter, and end with a digit, how many such passwords are there?

**Solution:** The number of combinations is given by \((52)(62)^5(10)\)

10. Texas Transportation Institute measured the wait at the red light at the corner of University and Texas Avenue. They found the following data:
<table>
<thead>
<tr>
<th>Wait (in sec.)</th>
<th>Number of Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t \leq 5$</td>
<td>50</td>
</tr>
<tr>
<td>$5 &lt; t \leq 10$</td>
<td>100</td>
</tr>
<tr>
<td>$10 &lt; t \leq 15$</td>
<td>150</td>
</tr>
<tr>
<td>$15 &lt; t \leq 20$</td>
<td>200</td>
</tr>
<tr>
<td>$20 &lt; t \leq 25$</td>
<td>100</td>
</tr>
<tr>
<td>$25 &lt; t \leq 30$</td>
<td>50</td>
</tr>
<tr>
<td>$30 &lt; t \leq 45$</td>
<td>30</td>
</tr>
<tr>
<td>$45 &lt; t \leq 60$</td>
<td>20</td>
</tr>
</tbody>
</table>

a) What is the probability that you will wait less than 25 seconds at the light?

**Solution:** We add up the number of cars waiting 25 seconds or less and get $50 + 100 + 150 + 200 + 100 = 600$. The probability is given by the relative frequency $600/700 = 6/7$.

b) What is the probability that you will wait between 10 and 20 seconds at the light?

**Solution:** We add up the number cars obtaining $150 + 200 = 350$. The probability is given by the relative frequency $350/700 = 1/2$.

11. A student casts a dice. If the dice is 4 or more, he casts another dice. If the second dice is 3 or less, he eats at Burger King. In every other case, he eats at MacDonalds.

Draw the tree diagram corresponding to this process.

**Solution:**

```
       M
       |
       M
       |
       M
     /|
    / | M
  /   |
 /     |
 ----|----
   M   M
     /|
    / | M
  /   |
 /     |
 ----|----
  BK   BK
     /|
    / | M
  /   |
 /     |
 ----|----
   M   M
     /|
    / | M
  /   |
 /     |
 ----|----
  BK   BK
     /|
    / | M
  /   |
 /     |
 ----|----
   M   M
```

**Extra Credit** A dart board is 1 meter in diameter. The bullseye is 4 cm wide. If every point on the board is equally likely, what is the probability of hitting the bullseye from 5 meters away?

**Solution:** The answer is given by the ratio of the area of the bullseye to the target. Therefore we have

\[
\frac{\pi(0.02)^2}{\pi(0.5)^2} = (0.04)^2 = 0.0016 = 0.16\%
\]