Exam # 3 - Solutions
Math 166.508
Fall 1999

Name: ______________________  Student ID: ______________________

Signature: ___________________

The following exam consists of 10 work-out problems worth a total of 100 points. Partial credit will be awarded according to completeness of work. Put the solutions in order on the blank sheets provided. Indicate clearly your answer and any work you wish to be considered for partial credit. There is a 5 point extra credit problem.

You may begin the exam when the instructor indicates.
1. [10 pts] Suppose \( P(A) = 0.6 \), \( P(B) = 0.4 \) and \( P(A \cap B) = 0.2 \). Then \( P(A^c \cap B^c) = \)

**Solution** From deMorgan’s Laws, \( A^c \cap B^c = (A \cup B)^c \), therefore \( P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1.0 - 0.6 - 0.4 + 0.2 = 0.2 \) This can also be seen from the figure below:

![Venn Diagram](image)

2. [10 pts] At a certain plant, 20% of workers can complete a task in less than 3 minutes, 60% can complete the task in 4 minutes or less, and 10% require more than 5 minutes. What is the probability that the worker can complete the task between 4 and 5 minutes?

**Solution** Notice that the intervals overlap (although this is not needed to solve the problem). Since 60% can finish in less than 4 minutes, and 10% finish in more than 5 minutes, it is clear that 30% = 100% - 60% - 10% take between 4 and 5 minutes.

3. [10 pts] An urn contains 3 red balls, 4 blue balls, and 5 black balls. 3 balls are taken from the urn, without replacement, what is the probability that they are different colors?

**Solution** There are \( C(3,1) \) ways to choose a red ball, \( C(4,1) \) ways of choosing a blue ball, and \( C(5,1) \) ways of choosing a black ball. There are \( C(3,1)C(4,1)C(5,1) \) ways of choosing one ball of each color. There are \( C(12,3) \) ways of choosing 3 balls (of any color) from the 12 in the urn. Therefore the probability is given by

\[
\frac{C(3,1)C(4,1)C(5,1)}{C(12,3)} = \frac{3 \times 4 \times 5}{11} = 0.2727
\]

4. [10 pts] A roulette wheel has 38 numbers \{0,00,1,2,...,36\}. What is the probability that an even number \{2,4,6,...,36\} appears 10 times in a row?

**Solution** There are 18 even numbers, \{2,4,6,...,36\}, out of 38 possible numbers, therefore the probability of getting an even number is \( \frac{18}{38} \) Consequently, the probability of getting 10 evens in a row is

\[
\left(\frac{18}{38}\right)^{10} = (0.4737)^{10} = 5.687 \times 10^{-4}
\]

5. [5 pts] A pair of fair dice is cast. What is the probability that the sum of numbers falling uppermost is less than 9, if it is known that one of the numbers is a 6?

**Solution** There are 11 combinations containing a 6:

\[
\{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (1,6), (2,6), (3,6), (4,6), (5,6)\}
\]

Four of these sum to less than 9: \( \{(6,1),(6,2),(1,6),(2,6)\} \). Therefore the conditional probability that the sum is less than 9, **given that one of the die is a 6** is given by

\[
\frac{4}{11}
\]
6. [10 pts] Urn A contains four white and six black balls. Urn B contains three white and five black balls. A ball is drawn from urn A and transferred to urn B. A ball is then drawn from urn B. What is the probability that the transferred ball is white, given the second ball was white? **Draw a tree diagram!**

**Solution** The tree diagram is shown below:

![Tree Diagram]

From Bayes’ Theorem, the probability that the transferred ball was white, given that the second ball was white is

\[
P(W_1|W_2) = \frac{(4/10)(4/9)}{(4/10)(4/9) + (6/10)(3/9)} = \frac{16}{34} = 0.4706
\]

7. The following data give the number of cars observed waiting in line at a drive-in teller between 3pm and 5pm on a certain Friday.

<table>
<thead>
<tr>
<th>Number of Cars</th>
<th>Frequency of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

a) [5 pts] Graph the data in the form of a histogram.
b) [10 pts] Find the mean, median and mode of this distribution.

**Solution** The mean is given by \( \mu = 3.08 \), the median is 3, and the mode is 2.
8. In an area where the average house price is $100,000.00 A realty company calculated the following data.

<table>
<thead>
<tr>
<th>Number of Houses Sold</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.02</td>
<td>.03</td>
<td>.05</td>
<td>.07</td>
<td>.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Houses Sold</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.16</td>
<td>.17</td>
<td>.13</td>
<td>.11</td>
<td>.09</td>
<td>.06</td>
<td>.03</td>
<td>.01</td>
</tr>
</tbody>
</table>

[10 pts] If a realtor averages 3% commission on each house, what is the average salary?
The mean number of houses sold is 18.09, therefore the average salary is $(18.09)(100,000)(0.03) = $54,270.

[Extra Credit 5 pts] Estimate the probability of earning more than $10,000.00 than the average salary?
To earn $64,270.00, one must sell 22 houses or more. The probability of this is given by $P(22) + P(23) + P(24) = 0.06 + 0.03 + 0.01 = 0.1$ This is the most exact answer.

Alternately, by Chebycheff, since the standard deviation is 2.60, this accounts for $7,812.62 = (2.60)(100,000)(0.03)$ $10,000.00 is therefore 1.28σ, and by Chebycheff, the probability of being within 1.28σ is greater than $1 - 1/(1.28)^2 = 0.3896$ The probability of being outside this range is less than $1 - 0.3896 = 0.6104$ Assuming the distribution symmetric, the probability of being greater than 1.28σ is less than 0.3052
Comparing this to the first method of solution, it is a tremendous overestimate of the probability.

9. Mr. Hunt is considering two business ventures. The anticipated returns (in thousands of dollars) is described by the following

<table>
<thead>
<tr>
<th>Venture A:</th>
<th>Venture B:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>Probability</td>
</tr>
<tr>
<td>-20</td>
<td>0.3</td>
</tr>
<tr>
<td>40</td>
<td>0.4</td>
</tr>
<tr>
<td>50</td>
<td>0.3</td>
</tr>
</tbody>
</table>

a) [5 pts] Which venture would provide Mr. Hunt with the higher expected return?

Solution The mean and standard deviation for Venture A are $\mu = 25$ and $\sigma = 29.75$ respectively. The mean and standard deviation for Venture B are $\mu = 24$ and $\sigma = 19.97$ respectively. Therefore A provides the higher prospective return.

b) [5 pts] Which investment involves the least risk (smallest standard deviation)? B has the smaller standard deviation.

Solution

10. [10 pts] A fair coin is tossed 100 times. What is the probability that exactly 50 heads occur?

Solution The answer is given by the binomial distribution function

$$P(100, 50) = C(100, 50)(1/2)^{50}(1/2)^{50} = 0.0796$$