What are the odds?

1. **Royal Flush.** This consists of a \( \{10, J, Q, K, A\} \) of the same suit. The number of ways that this can come about is (number of ways of choosing 1 suit from 4 suits) \( \times \) (number of ways to choose \( \{10, J, Q, K, A\} \)) = \( C(4,1) \times C(5,5) = 4 \times 1 = 4 \).

2. **Straight Flush.** This consists of 5 cards in a row of the same suit. The number of ways that this can come about is (number of ways of choosing 1 suit from 4 suits) \( \times \) (number of ways of choosing 5 cards in a row) = \( C(4,1) \times (10) = 40 \).

3. **Four of a Kind.** The number of ways that this can come about is (number of ways of choosing 1 rank from 13 ranks) \( \times \) (number of ways of choosing 4 cards from 4) \( \times \) (number of ways of choosing the fifth card) = \( 13 \times C(4,4) \times C(48,1) = 13 \times 1 \times 48 = 624 \).

4. **Full House.** This consists of three of a kind and two of a kind. The number of ways that this can come about is (number of ways of choosing 1 rank from 13) \( \times \) (number of ways of choosing 3 cards from 4) \( \times \) (number of ways of choosing 1 rank from 12) \( \times \) (number of ways of choosing 2 cards from 4) = \( C(13,1) \times C(4,3) \times C(12,1) \times C(4,2) = 13 \times 1 \times 12 \times 6 = 3,744 \).

5. **Flush.** This consists of 5 cards of the same suit, but not in sequence. The number of ways that this can come about is (number of ways of choosing 1 suit from 4) \( \times \) (number of ways of choosing 5 cards from 13 in a suit) - (number of straight flushes) = \( C(4,1) \times C(13,5) - 40 = 4 \times C(13,5) - 40 = 5,108 \).

6. **Straight.** This consists of 5 cards in a row, but not of the same suit. The number of ways that this can come about is (number of ways of choosing 5 values in a row) \( \times \) (number of ways of choosing first card from 4) \( \times \) (number of ways of choosing second card from 4) \( \times \) (number of ways of choosing third card from 4) \( \times \) (number of ways of choosing fourth card from 4) \( \times \) (number of ways of choosing fifth card from 4) - (number of straight flushes) = \( 10 \times C(4,1)^5 - 40 = 4 \times 10 - 40 = 10,200 \).

7. **Three of a Kind (but not full house, or four of a kind).** The number of ways that this can come about is (number of ways of choosing 1 rank from 13) \( \times \) (number of ways of choosing 3 cards from 4) \( \times \) (number of ways of choosing 1 card from remaining 48) \( \times \) (number of ways of choosing 1 card from remaining 47) - (number of four of a kind) - (number of full house) = \( C(13,1) \times C(4,3) \times C(49,1) \times C(48,1) - 624 - 3744 = 13 \times 4 \times 49 \times 48 - 624 - 3744 = 117,936 \).

8. **Two Pair (but not four of a kind).** The number of ways that this can come about is (number of ways of choosing two different ranks from 13) \( \times \)
(number of ways of choosing two cards from four) \times (number of ways of choosing two cards from four) \times (number of ways of choosing remaining card) = C(13,2) \times C(4,2) \times C(4,2) \times C(44,1) = 78 \times 6 \times 6 \times 44 = 123,552.

9. One Pair (but not two pair, or three of a kind or full house). The number of ways that this can come about is (number of ways of choosing 1 rank from 13) \times (number of ways of choosing two cards from four) \times (number of ways of choosing another rank) \times (number of ways of choosing one card from four) \times (number of ways of choosing another rank) \times (number of ways of choosing one card from four) \times (number of ways of choosing another rank) = C(13,1) \times C(4,2) \times C(12,1) \times C(4,1) \times C(11,1) \times C(4,1) \times C(10,1) \times C(4,1) / 3! = 13 \times 6 \times 12 \times 4 \times 11 \times 4 \times 10 \times 4 = 1,098,240.

The total number of 5 card combinations are \( C(52,5) = 2,598,960 \).
The probabilities are therefore:

1. Royal Flush. 4 in 2,598,960.
2. Straight Flush. 40 in 2,598,960.
3. Four of a Kind. 624 in 2,598,960.
8. Two Pair 123,552 in 2,598,960.
9. One Pair 1,098,240 in 2,598,960.