PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 5 points: NO partial credit will be given. The use of a calculator is prohibited.

1. Find the area of the region bounded between the curves $y = 0$ and $y = \sin x$ from $x = \pi/4$ to $x = \pi/2$.
   (a) 1
   (b) $\frac{\sqrt{2}}{2}$
   (c) $\sqrt{2}$
   (d) $\frac{\sqrt{3}}{3}$
   (e) $\sqrt{3}$

2. Find the average value of the function $g(x) = \sqrt{1+2x}$ on the interval $[1,4]$.
   (a) $9 - \sqrt{3}$
   (b) $18 - 2\sqrt{3}$
   (c) $6 - \frac{2\sqrt{3}}{3}$
   (d) $3 - \frac{\sqrt{3}}{3}$
   (e) $\frac{3}{2} - \frac{\sqrt{3}}{6}$

3. The ellipse $\frac{x^2}{4} + \frac{y^2}{36} = 1$ is revolved about the $x$-axis. Which integral gives the volume of the resulting ellipsoid?
   (a) $\pi \int_{-2}^{2} (36 - 9x^2) dx$
   (b) $\pi \int_{-6}^{6} (36 - 9x^2) dx$
   (c) $2\pi \int_{-2}^{2} x\sqrt{36 - 9x^2} dx$
   (d) $2\pi \int_{-6}^{6} x\sqrt{36 - 9x^2} dx$
   (e) $\pi \int_{-2}^{2} (36 - 9x^2)^2 dx$
4. Using a trigonometric substitution, the integral $\int \frac{x^2}{\sqrt{x^2 + 25}} \, dx$ becomes:

(a) $25 \int (\tan^2 \theta)(\sec \theta) \, d\theta$
(b) $5 \int (\tan^2 \theta)(\sec \theta) \, d\theta$
(c) $25 \int \frac{\tan^2 \theta}{\sec \theta} \, d\theta$
(d) $5 \int \frac{\tan^2 \theta}{\sec \theta} \, d\theta$
(e) $25 \int \sin^2 \theta \, d\theta$

5. Compute $\int_{0}^{4} \sqrt{16 - x^2} \, dx$

(a) 2
(b) 4
(c) 0
(d) $2\pi$
(e) $4\pi$

6. If $F(0) = 1$ and $F(3) = 5$, then $\int_{0}^{3} F'(x) \, dx =$

(a) 8
(b) 6
(c) 5
(d) 4
(e) Can’t be determined from the given information.
7. Which of these expressions represents the area between the curves \( y = x^2 \) and \( y = 6 - x \) from \( x = 0 \) to \( x = 3 \)?

(a) \( \int_0^3 (6 - x - x^2) \, dx \)

(b) \( \int_0^3 (x^2 + x - 6) \, dx \)

(c) \( \int_0^2 (6 - x - x^2) \, dx + \int_2^3 (x^2 + x - 6) \, dx \)

(d) \( \int_0^2 (x^2 + x - 6) \, dx + \int_2^3 (6 - x - x^2) \, dx \)

(e) \( \int_0^1 (6 - x - x^2) \, dx + \int_1^3 (x^2 + x - 6) \, dx \)

8. The base of a solid is the ellipse \( x^2 + \frac{y^2}{9} = 1 \). Cross-sections perpendicular to the \( y \)-axis are squares. Find the volume.

(a) \( \int_0^3 4 \left( 1 - \frac{y^2}{9} \right) \, dy \)

(b) \( \int_{-3}^3 4 \left( 1 - \frac{y^2}{9} \right) \, dy \)

(c) \( \int_{-3}^3 1 - \frac{y^2}{9} \, dy \)

(d) \( \int_{-1}^1 4 \left( 1 - \frac{y^2}{9} \right) \, dy \)

(e) \( \int_0^1 1 - \frac{y^2}{9} \, dy \)
9. Evaluate \( \int_0^1 xe^{-x} \, dx \)

   (a) 1
   (b) \( 1 + \frac{1}{e} \)
   (c) \( 1 - \frac{1}{e} \)
   (d) \( 1 + \frac{2}{e} \)
   (e) \( 1 - \frac{2}{e} \)

10. A water tank has the shape of a hemisphere with radius 5 meters. It is filled with water to a height of 2 meters. Find the work in Joules required to empty the tank by pumping all of the water to the top of the tank. Here, \( \rho \) is the density of water in kilograms/(meter)\(^3\) and \( g \) is the acceleration of gravity in meters/(second)\(^2\).

   (a) \( \frac{625\pi \rho g}{4} \)
   (b) \( \frac{52\pi \rho g}{3} \)
   (c) \( 64\pi \rho g \)
   (d) \( 36\pi \rho g \)
   (e) \( 72\pi \rho g \)
PART 2: WORK-OUT PROBLEMS

Each problem is worth 10 points; partial credit is possible. The use of a calculator is prohibited. SHOW ALL WORK!

11. Evaluate \( \int (\ln x)^2 \, dx \)

12. Compute \( \int_2^{2\sqrt{2}} \frac{\sqrt{x^2 - 4}}{x} \, dx \)
13. Consider the region $R$ bounded by the curves $y = 4 - x^2$ and $y = -3x$.

(a) Set up and evaluate an integral with respect to $x$ that gives the area of the region $R$. (6 pts)

(b) Find, but DO NOT evaluate an expression that involves integration with respect to $y$ that represents the area of the region $R$. (4 pts)
14. Consider the region $R$ bounded between the curves $y = x^3$ and $y = 2x^2$.

(a) Find the volume of the solid obtained by revolving the region $R$ about the $x$-axis. (5 pts)

(b) Find the volume of the solid obtained by revolving the region $R$ about the $y$-axis. (5 pts)
15. Evaluate $\int (\sec^5 x)(\tan^3 x) \, dx$